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Numerical-Aided Elasto-Plastic Model for Circular Tunnel in Hoek-Brown Rock Masses

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ABSTRACT

This paper presents an elasto-plastic analytical solution for an axi-symmetrical circular tunnel. On account of the mathematical complexity, numerical treatments have been used to assist the elasto-plastic solution in evaluating the stress equilibrium, the strain compatibility equation, and the radius of yielding zone. In proposed model, rock mass obeys the latest non-linear generalized Hoek-Brown yield criterion, developed in 2002, in terms of its peak and residual strength parameters. The proposed approach considers a variable value for and residual strength parameter of rock mass, as governed by a strength loss parameter. The strength loss parameter makes it possible to model either elastic-perfectly plastic or elastic- brittle-plastic behaviour of the rock mass. The solution results in the analysis of the stress and strain state and the representation of the Ground Reaction Curve (GRC), which is commonly used in convergence-confinement method.

1 INTRODUCTION

The Preliminary design of an underground structure such as tunnels and shafts requires an analytical solution to predict the stress and strain state. An ealsto-plastic solution makes it possible to determine the stresses, the displacements, and the radius of the plastic zone around the tunnel. For the assumption of the isotropy in field stress, circular shape of tunnel, homogeneity in rock mass, and axi-symmetrical plane strain condition, several approaches have been developed over the past 30 years, in which either Mohr-Coulomb (M-C) or Hoek-Brown (H-B) yield criterion was used.

Considering different models of rock behaviour, such as the elastic-perfectly plastic, elastic-brittle-plastic and elastic-strain softening, with different yield criteria M-C and H-B, the complexity of the solution differs.

Most of the straightforward solutions (Ladanyi 1974; Florence & Schwer 1978; John et al. 1984; Senseny et al. 1989; Indraratna & Kaiser 1990; Pan & Chen 1990; Panet 1993; Duncan Fama 1993) used the linear Mohr-Coulomb failure criterion whereas more complicated solutions (Hoek & Brown 1980, Brown et al. 1983; Ogawa & Lo 1987; Deournay & John 1988; Wang 1996; Cundall et al. 2003; Carranza-Torres & Fairhurst 1999; Carranza-Torres 2004; Sharan 2003, 2005,2007; Park & Kim 2006; Sofianos & Nomikos,2006; Park et al. 2008) were based on non-linear Hoek-Brown yield criterion.

Up to date, the most difficult part of deriving the equations was to obtain the plastic strain in plastic zone that necessitate many mathematical simplifications and treatments. More recently, researches have tried to develop an elasto-plastic solution that satisfies the latest version of H-B yield criterion (Hoek et al. 2002) in which $a \geq 0.5$. Among them, Carranza-Torres (Carranza-Torres 2004) proposed a comprehensive closed-form solution using a transformation technique that made the solution more complex. Recently, Sharan (2007) pointed out the errors in the solutions by Brown et al. (1983) and Wang (1996) and developed also a closed form solution for $a \geq 0.5$. However, the author has found this solution to be incorrect.

The objective of this paper is to develop a numerical-aided elasto-plastic solution for the analysis of the stress and strain state around a circular tunnel in an elastic-perfectly plastic or elastic-brittle-plastic rock media obeying non-linear generalized Hoek-Brown yield criterion with $a \geq 0.5$. The proposed solution utilizes simple mathematical treatments to alleviate the complexity of the problem. The available mathematical softwares namely Matematica (Matematica, Wolfram Research 2004) and Maple (Maple Inc 2003) are used to solve the stress equilibrium and strain compatibility equations. The resulting integration in product of the strain compatibility equation is evaluated by so-called Sympson's rule. The radial stress at elastic-plastic boundary around the tunnel is evaluated numerically using the well-known Newton-Raphson Method. All formulations of the solution can be implemented by a programmable calculator for quick usage.

2 DEFINITION OF THE PROBLEM AND THE MAIN ASSUMPTIONS

The problem is defined in Figure 1. Consider a deep circular tunnel being excavated in an infinite medium subjected to isotropic hydrostatic initial stress, P_o (K=1). The excavation removes the boundary stresses around the circumference of the tunnel, and the process may be simulated by gradually reducing the internal (support) pressure, P_i. As P_i is reduced, a plastic zone is formed when the material is overstressed, and the radial displacement, u_r, occurs. It is required to compute the stresses and displacements around the tunnel, when plane strain condition along the axis of the tunnel is reached. The assumptions of homogeneity, isotropy, time independency, and linear elasticity prior to failure of the rock mass are made. The rock mass strength is assumed to follow the non-linear H-B yield criterion (Hoek et al.2002). Elastic-brittle-plastic or elastic-perfectly plastic material models with a constant rate of dilation, followed by a non-associated flow rule of plasticity are simulated. The deformation pattern near the tunnel is properly described by a plane strain condition. A section of tunnel far from the face is considered so that the 3-D effects at the tunnel face are eliminated. Consequently, the proposed solution can be applied to predict the ultimate tunnel convergence, at least two tunnel diameters behind the face.

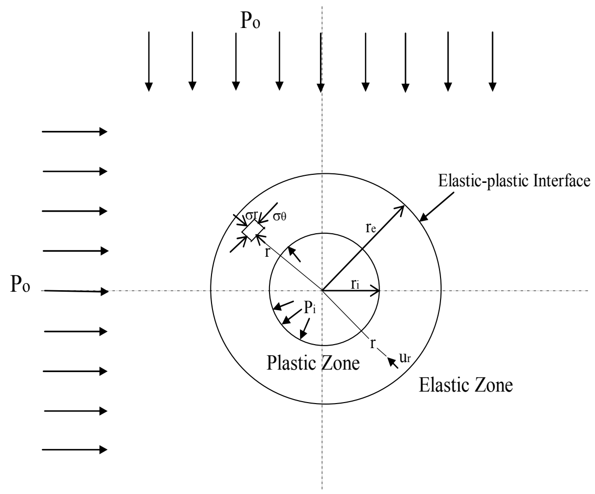

Figure 1: Definition of the model.

3 SOLUTION METHOD

3.1 Stress analysis

For a solution of the elasto-plastic problem, the equation of equilibrium, the compatibility condition, a stress-strain relationship in the elastic field, a yield criterion, a plastic potential, and a flow rule are required. The stresses and displacements in the elastic region can be easily determined by observing the continuity of radial stresses and displacements at the elastic-plastic interface. The solution within the plastic region will depend on the assumption of (a) the yield criterion, (b) the use of an associated or a non-associated flow rule, and (c) the dilatancy angle ψ.

Yield initiation is assumed to occur following a non-linear Hoek-Brown failure criterion. In this elasto-plastic solution, the latest version of the Hoek-Brown yield criterion (introduced in 2002) has been chosen:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a \tag{1}$$

The coefficients m_b, s and a in Equation 1 are semi-empirical parameters. In practice, these parameters are associated with the Geological Strength Index (GSI), which characterizes the rock mass (Hoek, 1994; Hoek & Brown, 1997). This index lies in a range of 5-85 and can be quantified from available qualitative or quantitative charts based on the degree of jointing of the rock structure and the condition of the discontinuities. The constants m_b, s and a of Equation 1 are in turn obtained by GSI (Geological Strength Index) (Hoek *et al.,* 2002):

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D} \right) \tag{2}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D} \right) \tag{3}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3} \right) \tag{4}$$

In Equations 2 and 3, D is a factor that depends on the degree of disturbance to which the rock has been subjected due to blast damage and stress relaxation. This factor varies between 0 and 1 (Hoek *et al*., 2002). The Hoek-Brown yield condition for post-peak (residual) strength parameters, used for the yielded zone around the excavation can be rewritten as:

$$\sigma_\theta = \sigma_r + \sigma_{ci}'\left(m_b' \frac{\sigma_r}{\sigma_{ci}'} + s' \right)^{a'} \tag{5}$$

where σ'_{ci} is the residual strength of the intact rock, m'_b, s' and a' are residual strength parameters of Hoek-Brown failure criterion. As it has been proved that the extension of the broken zone relies on the residual value of the intact rock strength (Hoek & Brown, 1980; Brown *et al*., 1983; Indraratna & Kaiser, 1990a; Cundall *et al*., 2003; Carranza-Torres, 2004), so the effect of the compressive strength of the rock material should be included in the form of the residual value as it loses its initial value due to stress relief or an increase in the strain. A stress reduction scale should, therefore, be considered as:

$$\sigma'_{ci} = S_r.\sigma_{ci} \quad (6)$$

where S_r refers to the strength loss parameter that quantifies the jump in strength from the intact condition to the residual condition or a measure of the degree of loss in strength that occurs immediately after the peak strength is reached. The parameter S_r characterizes the brittleness of the rock material: ductile, softening, or brittle. By definition, S_r will fall within the range $0 \le S_r \le 1$ where $S_r = 1$ implies no loss in strength and the rock material is ductile, or perfectly plastic. On the other hand, if $S_r=0$, the rock is brittle (elastic-perfectly brittle plastic) with the minimum possible value for the residual strength.

The combination of the stress equilibrium equation and Hoek-Brown failure criterion (Equation 5) results in a non-linear differential equation for the determination of the stress in the plastic (broken) zone around the tunnel:

$$\frac{d\sigma_r}{dr} - \frac{\sigma'_{ci}\left(m'_b \frac{\sigma_r}{\sigma'_{ci}} + s'\right)^{a'}}{r} = 0 \quad (7)$$

The solution of the above differential equation isobtained, taking into account the boundary condition (at $r=r_i$, $\sigma_r =0$):

$$\sigma_r = \frac{\Gamma - s'}{\frac{m'_b}{\sigma'_{ci}}} \quad (8a)$$

$$\Gamma = \left[s'^{1-a'} - m'_b(a'-1)\ln\left(\frac{r}{r_i}\right) \right]^{\frac{1}{1-a'}} \quad (8b)$$

Continuity of radial stress through the whole rock medium is assumed for the determination of the plastic zone radius. The radial stress at the elastic-plastic interface can be considered as a fictitious internal pressure for the outer elastic zone. In the pure elastic zone, the stress distributions are determined using so-called Lame's solution. Hence, the following non-linear equation must be solved to determine the plastic zone radius. This approach has already been discussed by Brown et al. (1983), Wang (1996), Osgoui (2007), and Sharan (2008).

$$2(P_o - \sigma_{re}) = \sigma_{ci}\left(m_b \frac{\sigma_{re}}{\sigma_{ci}} + s \right)^a \quad (9)$$

An exact solution is only possible when $a = 0.5$ as determined by Brown et al. (1983), Sharan (2003, 2005), Park & Kim (2006), and Park et al. (2008):

$$\sigma_{re,exact} = P_o + \frac{m_b\sigma_{ci}}{8} \pm \frac{1}{8}\sqrt{16P_o m_b\sigma_{ci} + m_b^2\sigma_{ci}^2 + 16\sigma_{ci}^2 s} \quad (10)$$

The negative sign in the above equation is acceptable and after abbreviating:

$$\sigma_{re,exact} = P_o - M\sigma_{ci} \quad (11a)$$

where:

$$M = \frac{1}{2}\left[\left(\frac{m_b}{4}\right)^2 + m_b\frac{P_o}{\sigma_{ci}} + s \right]^{\frac{1}{2}} - \frac{m_b}{8} \quad (11b)$$

On the other hand, a numerical technique; namely, the Newton-Raphson method (Press et al.2007), can be applied to approximate the exact solution of Equation 9 (Osgoui, 2006). If solved for $a \ge 0$, σ_{reN} is numerically calculated:

$$2(P_o - \sigma_{reN}) = \sigma_{ci}\left(m_b \frac{\sigma_{reN}}{\sigma_{ci}} + s \right)^a \quad (12)$$

By equating the radial stresses at the elastic-plastic interface, determined from both the elastic and plastic sides, the plastic zone radius r_e can numerically be determined by assuming continuity of radial stress at the elastic-plastic boundary. It is also assumed that the field boundaries are far enough from the tunnel such that their influence on the solution for re is negligible.

Equating σ_r of Equation 8a (for σ_{re} at $r=r_e$) and σ_{reN} of Equation 12, the normalized plastic zone radius can be derived as follows:

$$\frac{r_e}{r_i} = e^X \quad (13a)$$

$$X = \left[\frac{s'^{(1-a')} - \left(\sigma_{reN} \cdot \frac{m'_b}{\sigma'_{ci}} + s' \right)^{(1-a')}}{m'_b(a'-1)} \right] \quad (13b)$$

3.2 Strain analysis

Under the axi-symmetric plane strain condition, the strains and the displacements are expressed as (Timoshenko & Goodier, 1970):

$$u_r = u_r(r), u_\theta = 0, u_z = 0 \quad (14)$$

$$\varepsilon_r = \frac{du_r}{dr}, \varepsilon_\theta = \frac{u_r}{r}, \varepsilon_z = 0 \quad (15)$$

where the subscripts *r, θ,* and *z* denote the radial, tangential, and longitudinal (axial) directions, respectively.

The compatibility condition is given by (Timoshenko & Goodier, 1970):

$$\frac{d\varepsilon_\theta^t}{dr}+\frac{\varepsilon_\theta^t-\varepsilon_r^t}{r}=0 \qquad (16)$$

Strains in elastic zone

Hooke's law is applied to determine the radial and tangential strains in the elastic region surrounding the plastic zone for a plane strain condition (Timoshenko & Goodier, 1970):

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl} \quad , \quad i,j,k,l=1,2,3 \qquad (17a)$$

$$\varepsilon_r=\frac{1-\nu^2}{E}\left[\sigma_r-\left(\frac{\nu}{1-\nu}\right)\sigma_\theta\right]$$

$$\varepsilon_\theta=\frac{1-\nu^2}{E}\left[\sigma_\theta-\left(\frac{\nu}{1-\nu}\right)\sigma_r\right] \qquad (17b)$$

$$\gamma_{r\theta}=\frac{1}{G}\tau_{r\theta}$$

where the 81 components C_{ijkl} of the fourth-rank tensor C are material constants (stiffness matrix).

Substituting stresses in elastic zone, obtained by Lame's solution, into Equation 17 provides the strain field for plane strain condition:

$$\varepsilon_r=\frac{(1-2\nu)}{2G}P_o+\frac{(\sigma_{reN}-P_o)}{2G}\left(\frac{r_e}{r}\right)^2 \qquad (18a)$$

$$\varepsilon_\theta=\frac{(1-2\nu)}{2G}P_o-\frac{(\sigma_{reN}-P_o)}{2G}\left(\frac{r_e}{r}\right)^2 \qquad (18b)$$

Strains in plastic zone

For small deformation and infinitesimal strains, the total strains in the plastic zone are the sum of the elastic and plastic components:

$$\varepsilon^t=\varepsilon^e+\varepsilon^p \qquad (19a)$$

or in the polar coordinate:

$$\varepsilon_\theta^t=\varepsilon_\theta^e+\varepsilon_\theta^p$$

$$\varepsilon_r^t=\varepsilon_r^e+\varepsilon_r^p \qquad (19b)$$

where the superscripts *e* and *p* denote the elastic and plastic components, respectively. Hooke's law and flow rule have been applied to calculate the elastic and plastic strains, respectively. The elastic strains in the plastic zone are determined by substituting stresses in plastic zone into Hook's constitutive laws.

The plastic strains in the plastic zone are, instead, governed by an appropriate flow rule postulated for the yielding behaviour. The flow rule of plasticity relating the plastic strain increment $\dot{\varepsilon}^p$ to the plastic potential Q is given by (Hill, 1950; Brown, 1986):

$$\varepsilon^{\cdot p}=\lambda_f\frac{\partial Q}{\partial\sigma} \qquad (20)$$

Since the extent of yielding depends on the dilation characteristics of the failed rock, the flow rule must adopt the influence of dilation. In the present solution, a linear Mohr-Coulomb plastic potential has been adopted. Under a plane strain condition, the flow rule can be written as:

$$d\varepsilon_r^p+N_\psi d\varepsilon_\theta^p=0 \qquad (21)$$

where $N_\psi=\frac{1+\sin\psi}{1-\sin\psi}=\tan^2(45^\circ+\frac{\psi}{2})$ and ψ is the dilatancy angle of the rock.

Having been determined the elastic strains in the plastic zone, the combination of strain compatibility (Equation 16) with the flow rule (Equation 21) gives rise to a solution for the strain field in the form of a non-linear differential equation:

$$\frac{d\varepsilon_\theta^p}{dr}+\frac{\sigma'_{ci}}{2Gr}\left[s'^{1-a'}-m'_b(a'-1)\ln\left(\frac{r}{r_i}\right)\right]^{\frac{a'}{1-a'}}\begin{bmatrix}(1-2\nu)+\\ a'm'_b(1-\nu)\end{bmatrix}+\frac{1}{r}\begin{bmatrix}\varepsilon_\theta^p(1+N_\psi)+\\ \frac{\sigma'_{ci}}{2G}\left[s'^{1-a'}-m'_b(a'-1)\ln(\frac{r}{r_i})\right]^{\frac{a'}{1-a'}}\end{bmatrix}=0 \qquad (22a)$$

The tangential strain at the elastic-plastic boundary (at $r=r_e$) produced by the reduction of σ_r from its original value, P_o, to σ_{re} is (Brown et al. 1983; Sharan 2003, 2007; Park &Kim 2006):

$$\varepsilon_\theta=\frac{(P_o-\sigma_{reN})}{2G} \qquad (22b)$$

Hence, the solution of Equation 22a is obtained using software Matematica (Wolfram Research 2004) or Maple (Maple Inc 2003) as:

$$\varepsilon_\theta^p=\frac{1}{r^{1+N_\psi}}\begin{bmatrix}\frac{(P_o-\sigma_{reN})}{2G}r_e^{1+N_\psi}+\\ \left(\frac{\sigma'_{ci}}{2G}(1-\nu)(2+a'm'_b)\right)\cdot\\ \int_r^{r_e} r^{N_\psi-1}\left[s'^{1-a'}+m'_b(1-a')\ln\left(\frac{r}{r_i}\right)\right]^{\frac{a'}{1-a'}}dr\end{bmatrix} \qquad (23)$$

As can be observed from Equation 23, an integral function has been introduced into the result of the differential equation. The complete solution can be obtained provided that the integral on the right side of Equation 23 is evaluated numerically. Sympson's rule is applied to approximately solve the integration (Waner & Costenoble, 2006).

The radial displacement field can fainally be evaluated from any of the expressions of Equation 15, neglecting elastic strain due to its very small magnitude compared to the plastic strain ($\varepsilon_\theta^e << \varepsilon_\theta^p$) and substituting $r=r_i$. Therefore, the radial inward displacement of the tunnel surface can simply be determined as:

$$\frac{u_{ri}}{r_i} = \frac{1}{2G}\left[\begin{array}{l} \dfrac{(P_o - \sigma_{reN})}{r_i^{1+N\psi}} r_e^{(1+N\psi)} + \\ \dfrac{1}{r_i^{(1+N\psi)}} \sigma'_{ci}(1-\nu)(2+a'm'_b)\cdot \displaystyle\int_{r_i}^{r_e} r^{N_\psi - 1}\Gamma^{a'}dr \end{array}\right] \quad (24)$$

4 PRACTICAL APPLICATION OF THE PROPOSED MODEL

The following example, posed by Hoek & Brown (1980) and Carranza- Torres (2004), are intended to illustrate the practical application of the proposed solution and to compare the results of proposed model with those of Carranza-Torres's solution. The geomechanical parameters of the rock mass used are given in Table 1.

A comparison between the proposed solution and that which developed by Carranza-Torres (2004), in terms of stresses and displacements around the tunnel, has been made as demonstrated in Figures 2 and 3. As can be observed, a good agreement between both results is obtained. In spite of equality in radius of plastic zone (5.09m) for both solutions, the radial displacement at tunnel boundary by Carranza-Torres' solution are slightly higher that that of proposed elasto-plastic model. This can be attributed to the different ways in solving the strain compatibility equation resulting different values. The radial displacement at tunnel boundary are calculated 30.7 mm for proposed model and 34.1 mm for Carranza-Torres' solution.

The proposed elasto-plastic solution makes it possible to depict the Ground Reaction Curve (GRC), which is main component of the Convergence-Confinement Method in tunnel design. The GRC for geomechanical parameters given in Table 1 is presented in Figure 4.

Table 1. Input parameters used in the practical example

Geomechanical parameters		value
Tunnel radius r_i(m)		2
Far field stress P_o (MPa)		15
Deformation modulus E (GPa)		5.7
Poisson's ratio ν (-)		0.3
Hoek – Brown rock mass strength constant	m_b	1.7
	s	3.9E-3
	a	0.5
	m_b'	0.85
	s'	1.9E-3
	a'	0.5
σ_{ci} (MPa)		30
σ_{ci}' (MPa)		27
Dilatancy angle ψ (°)		0 (non-associated)
Dilatancy Paramerer $N\psi$		1

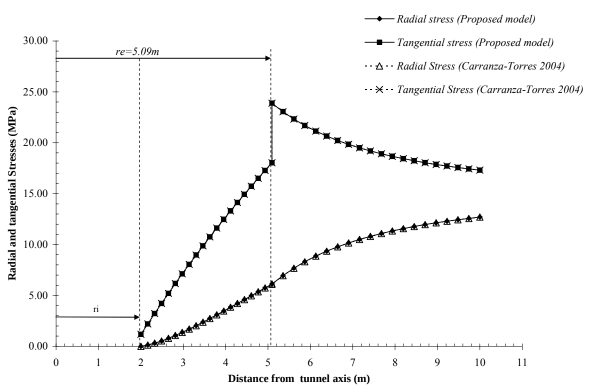

Figure 2: Distribution of the tangential and radial stresses calculated by proposed and Carranza-Torres' solutions.

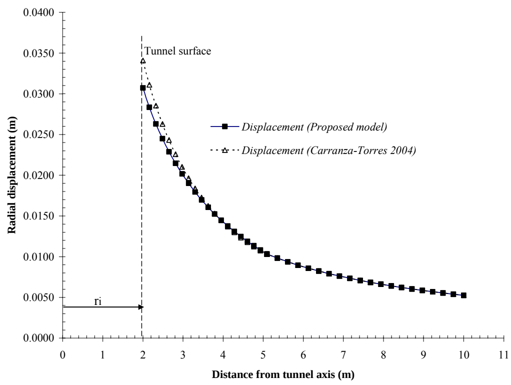

Figure 3: Displacement field obtained from both proposed and Carranza-Torres' solutions.

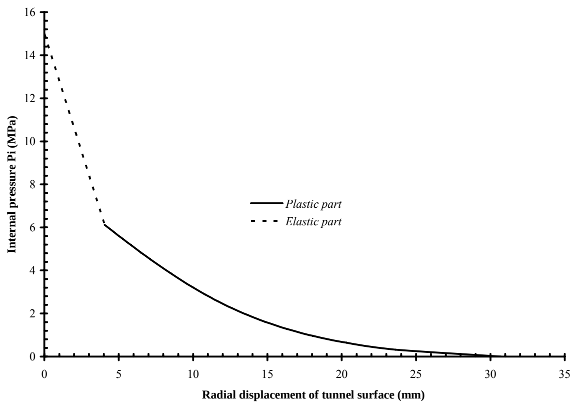

Figure 4: Ground Reaction Curve (GRC) based on the proposed elastic- plastic model

5 CONCLUSIONS

A numerical-based elasto plastic solution for an axi-symmetrical circular tunnel in an isotropic and homogeneous medium that obeys generalized Hoek-Brown failure criterion was developed. Various numerical techniques have been adopted in order to solve the equilibrium and compatibility equations apart from taking advantage of available mathematical programs. The essential aim of proposed solution is intended to predict the stress and strain states and extension of yielding around tunnel subjected to a hydrostatic stress field. Furthermore, the proposed solution allows the representation of the Ground Reaction Curve (GRC), which is considered as a practical means in tunnel support design. The proposed solution is also capable of predicting the ultimate tunnel convergence (at least two tunnel diameters behind the face), where three dimensional face effect are ignored. The practical application of the proposed solution was presented using a real example. A comparison between the results of the proposed solution and those of Carranza-Torres has been carried out and a good agreement was acquired. The proposed solution provides a practical means to quickly evaluate the deformational behaviour of a tunnel. All calculation steps can simply be implemented by using a programmable calculator or spread sheet.

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NOMENCLATURE

GSI = Geological Strength Index
a= strength constant of Hoek-Brown failure criterion
a' = residual strength constant of Hoek-Brown failure criterion
c = cohesion of rock mass
D = disturbance factor of Hoek-Brown failure criterion
E = Young's (elasticity) modulus
G = shear modulus
k =stress ratio
m_i = strength constant of Hoek-Brown failure criterion for intact rock
m_b = strength constant of Hoek-Brown failure criterion
m_b' = residual strength constant of Hoek-Brown failure criterion
m_b^* = equivalent strength constant of Hoek-Brown failure criterion
N_ψ = dilation coefficient
P_i = fictitious support pressure
P_o = in-situ stress
Q = plastic potential
r = distance from tunnel center to point of interest
r_e = radius of plastic (broken, yielding) zone
r_i = tunnel radius
S_r =post-peak strength reduction factor
s = strength constant of Hoek-Brown failure criterion
s' = residual strength constant of Hoek-Brown failure criterion
s^* = equivalent strength constant of Hoek-Brown failure criterion
u_r = radial displacement
u_{ri} = displacement at tunnel surface
u_z = longitudinal displacement
γ = rock mass unit weight
$\gamma_{r\theta}$ =shear strain in axi-symmetric problem
ε_1 = maximum principal strain
ε_3 =minimum principal strain
ε_r = radial strain
ε_θ =tangential strain
ε_z =longitudinal strain
ε^e =elastic strain
ε^p =plastic strain
ε_θ^t = total tangential strain
ε_θ^e = elastic tangential strain
ε_θ^p = plastic tangential strain
ε_r^e = elastic radial strain
ε_r^p = plastic radial strain
ε_r^t = total radial strain
$\dot{\varepsilon}^p$ = incremental plastic strain in flow rule
λ_f = non-negative constant of proportionality in flow rule
ν = Poisson's ratio
σ = yield function
σ_1 = maximum principal stress
σ_3 = minimum principal stress
σ_{ci} = uniaxial compressive strength of intact rock
σ_{ci}' = residual compressive strength of intact rock
σ_r = radial stress
σ_θ = tangential stress
σ_{re} = radial stress at elastic-plastic interface
$\sigma_{re,exact}$ = exact solution of radial stress at elastic-plastic boundary
σ_{reN} = approximate (numerical) solution of radial stress at elastic-plastic boundary
$\sigma_{\theta e}$ = tangential stress at elastic-plastic interface
$\tau_{r\theta}$ =shear stress in axi-symmetric problem
ϕ = internal friction angle of rock
ψ = dilatancy angle of rock

Subscripts

r = radial
t = tangential

Superscripts

e = elastic
p =plastic
t = total
$\dot{}$ = increment