This technical article has been submitted to Rock Mechanics' research and development (R&D) of mining engineering department of Middle East Techical University as an advanced study.

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Summer 2003, Ankara- Turkey

ANALYTICAL DESIGN OF ROCK-BOLT SYSTEMS

"Imagination is more important than knowledge"
Albert Einstein

1.INTRODUCTION

Some forty years ago, most opening supports and reinforcements were designed using empirical rules defining ground loads acting on the supporting structures. This approach is still popular and interest in it has been renewed by the use of rock mass classification systems (Barton *et al*. 1974, Bieniawski, 1974,1976,1989). Correlations between rock mass conditions and the type of support used are based on case histories. This approach perpetuates existing practice, even if over-conservative or unsatisfactory.

Most recent underground excavation design relies on more elaborate analysis of the complex rock –support interaction. Moreover, the effect of geometry of opening has to be allowed for. A new concept of RRE (Rock-mass, Rock-bolt, Excavation) which will be developed in thesis, is intended to cover all parameters involving in design of a proper reinforcement system. This concept must take into account not only the properties of rock mass, but also the structure behavior of the reinforcing structure (element) and the opening geometry. Stresses and displacements in the rock mass surrounding openings depend upon the rock mass properties, the *in-situ* initial stress field, opening shape, and the type (stiffness) of the support and the time of its installation. In this study, the time –independent effect is disregarded.

A complete analysis requires a larger amount of data and information, which may be taken into account by numerical modeling. However, in most cases it is difficult to obtain all necessary data before construction. Many of the parameters that influence the analysis are ill-defined and, up to now, no perfect constitutive model has been shown to be successful at simulating all the aspects of rock behavior important to design of reinforcement systems for underground excavations. Some rock properties may only be evaluated by back-analyzing field data.

A good understanding of the deformations (the most dangerous phenomenon) caused by an underground opening requires simultaneously knowledge of the rock-support interaction and interpretation of field data. Assessing the stress acting on reinforcement system as well as displacements occurring during and after construction is the main purpose of analytical methods in designing a proper reinforcement system for defined opening. The measurement of the wall's displacements, commonly called " convergence" is indeed the one most used on a opening site.

2.GENERAL REMARK

In this stage of design of rock-bolt systems, an extensive literature survey conjunction with analytical models of passive rock-bolts is fully discussed. To date, a large number of analytical solutions have been obtained for rock-bolt design. It is clear that rock mass reinforcement techniques, such as by means of fully grouted bolts or cables, have been applied to openings, but theoretical problems have been encountered in the design of this reinforcement scheme with analytical axisymmetric models due to the difficulty of considering the influence of the passive bolt and the rock mass rock-bolt interaction (RR from RRE concept). Analytical axisymmetric models that can define the stresses and deformation induced by excavation (convergence-confinement method) are widely used in the design of excavation and reinforcement system due to the computational simplicity. Many analytical formulations for the calculation of the ground characteristics curve have been developed and a number of models, based on either the Mohr-Coulomb or the Hoek-Brown strength criterion with elastic-plastic or elastic-brittle plastic stress-strain behavior have been adopted. Some models with a strain softening behavior with non-linear strength criteria, and associated or non-associated flow rules have also been presented.

Table 1.Material (rock mass) behavior models used in rock-bolt design.

Author / Year	Strength/Yield Criterion	Stress-Strain Model	Treatment of plastic volumetric strains
Indraratna & Kaiser 1987,1990	Mohr-Coulomb (linear)	elastic-brittle plastic	non-associated flow rule
Peila & Oreste 1995,1996,1997, 2003	Hoek & Brown (non-linear)	strain-softening	associated flow rule applied over limited range of post peak strain
Hoek & Brown 1980,1983	Hoek & Brown(non-linear)	elastic-brittle plastic	associated flow rule applied over limited range of post peak strain
Stille et al. 1983,1989	Mohr-Coulomb (linear)	elastic-brittle plastic	non-associated flow rule dilatancy after failure

In order to better understand of rock-bolt and rock mass interaction (RR of RRE Concept), it is essential to get a good sense of material (rock) behavior, by means of which some important analytical design approaches have been developed (see Tabale1). The material behavior models used in analytical solution of rock-bolt design are illustrated in the Figure1 and 2.

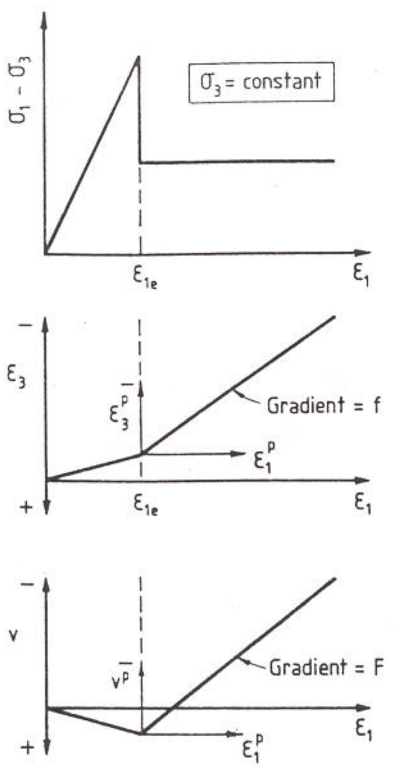


Figure1. Material behavior model in the case of Elastic-brittle plastic (after Brown *et al*. 1983).

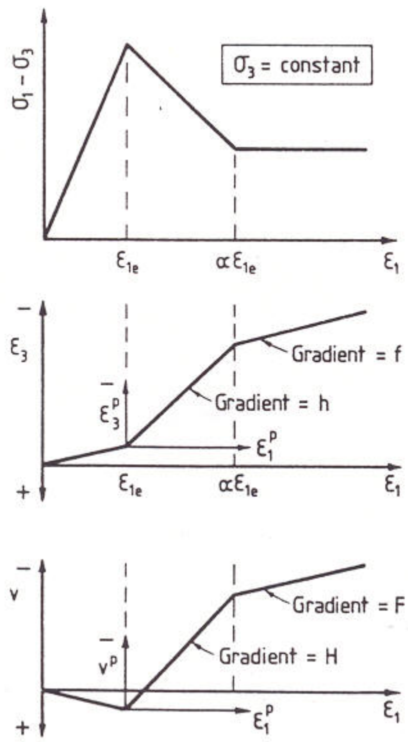


Figure2. Material behavior model in the case of the strain -softening (after Brown *et al*. 1983).

In general, rock reinforcement with different types of rock-bolts can be assessed with analytical solutions based on the theories of the ground reaction curve (Hoek & Brown, 1980, Brown *et al.* 1983, Stille *et al.*1989, Oreste & Peila, 1995,1996,1997,2003).

The arising deformations and stresses around an excavation in rock are a result from the interaction between rock mass and rock support. Analytical solution can be achieved under certain conditions. The problem is often solved for a circular opening in a hydrostatic stress field and presented in a ground-support interaction diagram with the ground reaction curve and the reaction line of the support (see Figure 3).

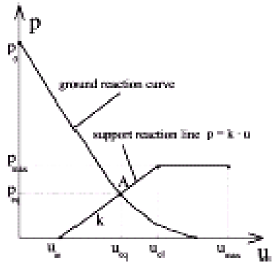


Figure 3. The convergence–confinement method. Key: *p*: internal opening pressure; *u*: radial displacement of the wall (positive towards the opening axis); p_o : *in situ* hydrostatic stress; p_{eq} : pressure acting on the support structure; p_{max} : pressure that induces the plastic failure of the structure (support capacity); *k*: support stiffness; u_{in} : displacement of the wall before support installation; u_{eq} : displacement at equilibrium; u_{el} : displacement of the wall on reaching the elastic el limit in the support; u_{max} : displacement of the wall on collapse of the max support; and *A*: equilibrium point of the opening-support system.

Many approaches to the calculation of the ground reaction curves for different types of rock mass behavior (see Table 1) have been reported in the literature. Brown, Bray, Ladanyi and Hoek (1983) have compiled a comprehensive report on the subject. They presented a stepwise method based on finite difference method of solving the partial differential equation for strain softening behavior of rock mass apart from normal method, which had been introduced earlier (Hoek & Brown, 1980).

Here let's briefly argue about grouted (in general name friction bolt) and prestressed types of bolt, which have been recently a debatable subject. The reaction line of a rock support with anchor bolts has been presented elsewhere(Hoek and Brwon 1980, Oreste 2003). The anchor bolt is connected to the rock in the anchoring zone and at the anchor head. Between these two connection points the rock is free to deform independently of the bolt. The most common anchor types are mechanical, cement grout, and chemical. The load in the rock-bolt is transferred to the opening surface at the anchor head through a nut and a washer plate.

The reaction line for a support with anchor bolts cannot be used for grouted bolts since the grouted bolts don't act independently of the rock. The deformations in the rock cannot be separated from the deformations in the bolt since the bolt and rock is grouted together. In such a cases the bolts will interact with the rock and influence the ground reaction curve of the rock mass.

Over the years, on the other hand, there has been much debate as to whether tensioned bolts or untensioned grouted bolts are more suitable for stabilizing rock openings. The question of whether to use tensioned bolts or untensioned grouted bolts is not of minor importance. Except from economic point of view, tensioned bolts are commonly used all over the world with the explanation that they are recommended in the empirical design method and will give the most efficient. It is doubtful whether this will in fact be achieved. The Q-system (Barton *et al*. 1974), for example, is such as an empirical method based on roughly 200 case records and therefore can be conservative. It reflects traditional support methods for advanced opening construction technique, which are not always the optimal ones. The Q-system recommends mainly the use of tensioned bolts for poorer rock qualities and untensioned bolts only for good and very good rock.

On the other side, in the Q-system, for many ground categories, particularly in poor, yielding rock it does not generally recommend the installation of untensioned grouted bolt or sweelex bolt. Therefore, it is not meaningful to compare the proposed analytical approached with the Q-system. The geomechanical classification or rock mass rating (RMR) developed by Bieniawski (1974,1979,1989) is, however, applicable to fully grouted bolts in all types of rock; accordingly: the analytical approaches in some cases can be compared with empirical reinforcement design.

Swidesh experience of rock-bolting today is that almost without exception untentioned grouted bolts and swellex bolts are utilized for every type of rock condition. Some technical report conducted by others in Sweden underlined the good experience from support systems using untensioned grouted bolts and shotcrete in poorer rock. Therefore, the behavior of grouted bolts as well as swellex bolts will mainly be analyzed in the thesis.

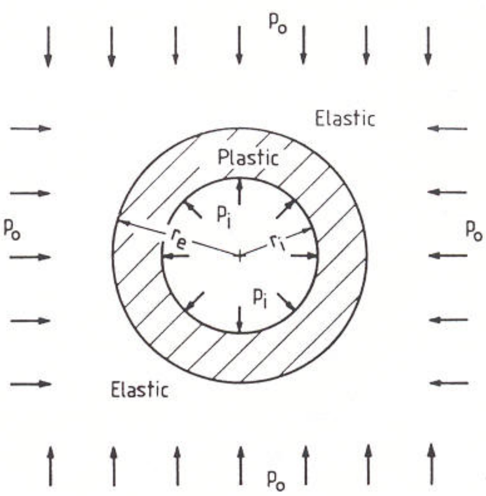
In the following, four constitutive models developed for the design of rock-bolt system based on analytical methods will be discussed. In is of great importance to utterly acquaint with those models involving the rock-bolts, rock mass interaction.

3. AXISYMMERTIC OPENING PROBLEM

Many authors have calculated the ground reaction curve of the rock mass for different failure criteria of the rock (Brown *et al.* 1983). For an elastic- brittle-plastic rock mass material with a Coulomb's failure criterion for both the peak and residual strength and a non-associated flow rule in which post-peak dilatancy occurs at a constant rate with major principal strain, the following solutions of the ground reaction curve will be achieved. This solution has been employed both Stille *et al.* (1983,1989) and Indraratna & Kaiser (1987,1990) (see Figure 4).

The stress-strain relation of the rock mass is illustrated in Fig.1. In the plastic (yielding or broken) zone nearest to the opening surface of the circular opening the following tangential stresses, σ_t, and radial stresses, σ_r, will occur:

Figure4.The axisymmertic opening problem



$$\sigma_t = (p_i + a_r)k_r\left[\frac{r}{r_i}\right]^{k_r - 1} - a_r \quad (1)$$

$$\sigma_t = (p_i + a_r)\left[\frac{r}{r_i}\right]^{k_r - 1} - a_r \quad (2)$$

where:

$$a_r = \frac{c_r}{\tan\phi_r},$$

$$k_r = \tan^2\left[45 + \frac{\phi_r}{2}\right],$$

c_r= residual value of the cohesion,

φ_r= residual value of the friction angle,

r_i= radius of opening,

p_i= outward radial pressure on the opening surface.

The boundary, r_e, between the zones of plastic and elastic behavior can for a rock mass with initial state of stress p_o be calculated with the following expression:

$$\frac{r_e}{r_i} = \left[\frac{\left[\frac{2}{1+k}(p_o + a) - a\right] - a_r}{p_i + a_r}\right]^{\frac{1}{k_r - 1}} \quad (3)$$

Above equation can also define the extent of the yielding zone where:

$$a = \frac{c}{\tan\phi} \ ,$$

$$k = \tan^2\left[45 + \frac{\phi}{2}\right],$$

c= cohesion,

φ= friction angle.

The stress outside the plastic zone (i.e. elastic zone), $r>r_e$, can be achieved with the following equations:

$$\sigma_t = p_o - (\sigma_{re} - p_o)\left[\frac{r_e}{r}\right]^2, \quad (4)$$

$$\sigma_r = p_o + (\sigma_{re} - p_o)\left[\frac{r_e}{r}\right]^2, \quad (5)$$

where:

$$\sigma_{re} = \frac{2}{1+k}(p_o + a) - a \quad (6)$$

σ_{re} is the radial stress at the elastic-plastic boundary.

The deformations of the opening surface, u_i, can be computed as:

$$u_i = \frac{r_i A}{f+1}\left[2\left(\frac{r_e}{r_i}\right)^{f+1} + (f-1)\right], \quad (7)$$

where

$$A = \frac{1+\nu}{E}\left(p_o - \sigma_{re}\right), \quad (8)$$

and

E= Young's (elasticity) modulus of rock mass

v=Poisson's ratio of rock mass,

p_o= *in-situ* stress

the factor f expresses the volume expansion after failure and is given by:

$$f = \frac{\tan\left(45 + \frac{\varphi}{2}\right)}{\tan\left(45 + \frac{\varphi}{2} - \psi\right)}, \quad (9)$$

where ψ= dilatancy angle.

Equation (7) is derived from the approximation that the rock mass is subjected to elastic strains in the plastic zone that are constant and equal to the elastic strains at the boundary between the zones of plastic and elastic behavior.

4.FLOW RULE OF PLASTICITY CONCEPT

No analytical model cannot introduce the real interaction between rock–bolt and rock mass around a opening provided that the real behavior of them is realized. At the failure point and post –peak behavior of a rock, it is important to determine the post-peak parameters of rock due to their applicability the analysis of the broken zone deduced as well as reinforced rock mass. Flow rule is recognized to be a tool, whereby; post peak parameters of rock can be determined based on selective yield condition. In what follows, we will assume that elastic stress increments have been computed and that both yield conditions are exceeded; later on, the condition for separate yielding will be given. The usual assumption is made that the overall strain increment of an element can be decomposed into elastic and plastic parts and, further, it is assumed that the plastic contributions of shear and volumetric yielding are additive. The principal axes of both plastic and elastic strain increment are taken to be coaxial with the principal axes of stress (only valid if elastic shear strains are small compared to plastic strains); the strain increments obtained are:

$$\Delta e_i = \Delta e_i^{\,e} + \Delta e_i^{\,ps} + \Delta e_i^{\,pv} \quad (10)$$

Where $i=1,3$ and superscripts ^{ps}and^{pv} stand for "plastic shear" and "plastic volume", respectively. The flow rules for shear and volumetric yielding are:

$$\Delta e_i^{\,ps} = \lambda_s \frac{\partial g_s}{\partial \sigma_i} \qquad \text{and} \qquad \Delta e_i^{\,pv} = \lambda_v \frac{\partial f_v}{\partial \sigma_i} \quad (11)$$

It can be shown that the volumetric flow rule is associated and the shear flow rule is non-associated. The plastic potential for shear yielding is:

$$g_s = \sigma_1 - \sigma_3 N_\psi + 2c\sqrt{N_\psi} \quad (12)$$

where $N_\psi = (1+\sin\psi)/(1-\sin\psi)$, and

ψ is the dilatancy angle.

An alternative approach to the use of experimentally determined parameters in defining the post-peak volumetric strain is to estimate them using the associated flow rule of the theory of plasticity. Hoek and Brown (1980) have previously used this approach in their closed from-solution. When an associated flow rule applies, the yield criterion and the plastic potential function are the same functions of the stress components. In other words, the flow rule is referred to as associated if the plastic potential and yield surface coincide. As a consequence of this, the plastic strain increment vector must be normal to the yield surface. If the yield surface is represented by a relation between principal stresses, σ_1 and σ_3, then the corresponding components of the strain increment vector are the increments of $\varepsilon_1^p$and ε_3^p. If the flow rule is non-associated, the yield criterion and the plastic potential function are not the same and the normality principles do not apply. There is limited evidence available to suggest that the dilation rate at peak stress in dense brittle rocks or tightly interlocked aggregates can be predicted closely using the associated flow rule. It is not clear, however, that the associated flow rule applies to heavily fractured and poorly interlocked rock masses. Indeed, analyses of data obtained from Brown *et al.*(1983) suggest that, in some such cases, the flow rule will be non-associated. This means that the resulting plastic volume changes will be less than those predicted using an associated flow rule.

More recently, based on analyses undertaken by Indraratna (1987), a flow rule applicable to a linear Mohr-Coulomb failure criterion, has been adopted:

$$\varepsilon_r^{\,p} + \alpha\varepsilon_\theta^{\,p} = 0 \quad (13)$$

The parameter α is the dilation coefficient that characterizes the volume change in the plastic zone. Zero volumetric strain (no volume change) is represented by $\alpha=1$. If $\alpha=m$, the associated flow rule is obtained for a Mohr-Coulomb material where $m=\tan(45+\ \varphi/2)$. For a material with a friction angle of 30°, a value of $\alpha=3$ is a upper bound. The associated low assumes that the plastic strain increments are normal to the failure envelope (2-D problem) satisfying normality condition, thereby generally overestimates the plastic strains in rock. Therefore, a non-associated flow rule

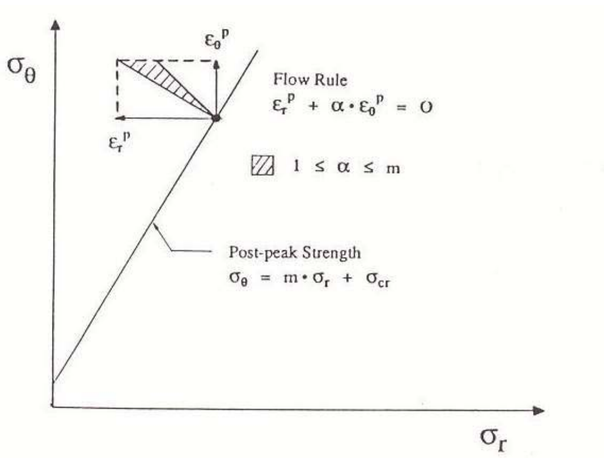


Figure 5. Mohr-Coulomb linear failure criterion and flow rule

(1<α<m) is more realistic, as illustrated by Figure 5.

A further attention has to be taken in differences between brittle material and plastic material (i.e. ideal plastic such as clay). Only for $1<\alpha<3$ the non associated flow rule satisfies the brittle material because for a brittle rock, $\alpha=3$ is a upper bound realized by non-associated flow rule whereas for $\alpha>3$ and $\alpha=1$ a plastic behavior occurs satisfied by a associated flow rule. A good definition for these hypotheses was studied by Peila *et al*. (1995) and Cividini (1993).

5. ANALYTICAL APPROACHES FOR ROCK-BOLT DESIGN

In this part of study, some crucial methods employing analytical methods for rock-bolt design conducted by others are extensively argued so that a complete and extensive literature on subject will be achieved. It is so convincing, at this point, that such constitutive models of rock-bolt design are mainly divided into two categories based on material behavior model and failure criteria.

PART A: LINEAR CONSTITUTIVE MODEL USING NON-ASSOCIATED FLOW RULE AND MOHR-COULOMB YIELD CONDITION

5.1. STILLE'S CONSTITUTIVE MODEL

Stille *et al.* (1983,1989) presented a closed-form elastic-plastic solution of grouted bolts by considering five different approaches of bolt performance which, even though introducing some simplifying assumptions, have proved to be in good agreement with measured data. In Stille's approach, the analysis of ground reaction curve for a rock mass with grouted rock-bolts is considered. In order to model the rock-bolt and rock mass behavior five following categories are taken into account.

a) *Elastic condition*
b) *Elastic bolt in a plastic rock mass*
c) *Plastic bolt in a plastic rock mass*
d) *Plastic deformation of the grouting material*
e) *Elastic bolt with nut and end-plate and plastic deformation of the grout material*

- **Elastic condition**

In the ***elastic condition*** grouted bolt improves the rock mass in terms of deformation modulus and stiffness. With the condition that the rock and the bolt will have the same strain the effective modulus E_{eff} can be obtained according to the theory of composite material and will get the following:

$$E_{eff} = E_b + \frac{E_s \cdot Y}{S_c \cdot S_l} \quad (14)$$

where E_b and E_s are the modulus of the rock mass and steel respectively and Y is bolt area. The bolt load T can be calculated from the following system of equations:

$$\varepsilon_r = -\frac{\partial u}{\partial r} \quad (15a)$$

$$u = \frac{1+\nu}{E_{eff}} \cdot (\sigma_{re} - p_o) \cdot \frac{{r_e}^2}{r} \quad (15b)$$

$$T = Y \cdot E \cdot \varepsilon_r \quad (15c)$$

The load will then be:

$$T = \frac{1+\nu}{E_{eff}} \cdot (\sigma_{re} - p_o) \cdot (\frac{r_e}{r})^2 \cdot E_s \cdot Y \quad (15d)$$

- **Elastic bolt in a plastic rock mass**

In the case of the ***elastic bolt in a plastic rock mass*** it is assumed that bolt distances are so small and rock mass and rock-bolt have the same strain, furthermore the occurrence of a tensile load in the bolt will imply a corresponding additional compressive stress in the rock mass. Radial stress in the rock mass will then be the sum of the outer stress, σ_r, and the above mentioned additional compressive stress. The equilibrium partial differential equations in the polar coordinate will occur:

$$\sigma_t - \sigma_r - r\frac{\partial \sigma_r}{\partial r} = 0 \quad (16)$$

$$\sigma_b = \sigma_r - \frac{T}{S_l S_c} \quad (17)$$

$$k_r = \frac{\sigma_t + a_r}{\sigma_b + a_r} \quad (18)$$

$$\frac{T}{Y} = E_s \varepsilon_s \quad (19)$$

Equation 16 is the condition of equilibrium for an infinitesimal element; equation 18 is the Mohr-Coulomb failure criterion for a rock mass with residual values on cohesion and friction angle. Equation 19 is the bolt stress from the plastic strain given by:

$$\varepsilon_s = \varepsilon_r - \varepsilon_{el} \quad (20)$$

Where the elastic strain, ε_{el} is assumed to be constant in the plastic zone and equal to:

$$-\varepsilon_{el} = A = \frac{1+\nu}{E}(p_o - \sigma_{re}) \quad (21)$$

Equation 21 is equal to the elastic strain at the boundary, $r=r_e$, between the zones of elastic and plastic behavior. The radial deformations, u_r, and the total radial strain, ε_r, depend on the properties of the rock mass and are obtained from:

$$u_i = \frac{r_i A}{f+1}\left[2\left(\frac{r_e}{r_i}\right)^{f+1} + (f-1)\right], \quad (22)$$

$$\varepsilon_r = -\frac{\partial u}{\partial r} \quad (23)$$

By combining equations (20-23) and 19 the bolt load, T, can be eliminated from equation17. The stress in the rock, σ_b, can then be omitted with equation18. Therefore, the tangential stress, σ_t, can be eliminated from equation 16 and the following partial differential equation will result:

$$r\frac{\partial \sigma_r}{\partial r} + \sigma_r(1-k_r) = C_3\left[\frac{r_e}{r}\right]^{f+1} - C_4 \quad (24)$$

This differential equation will have the following solution:

$$\sigma_r = C_5 r^{-(1-k_r)} - \frac{C_3}{f+k_r}\left[\frac{r_e}{r}\right]^{f+1} - \frac{C_4}{1-k_r} \quad (25)$$

The C_5 can be determined from the boundary condition in the way that the radial stress, $\sigma_r = p_i$, at the opening surface, $r=r_i$. The radial stress will then be:

$$\sigma_r = \left(p_i + \frac{C_4}{1-k_r} + \frac{C_3}{f+k_r}\left[\frac{r_e}{r_i}\right]^{f+1}\right)\left(\frac{r}{r_i}\right)^{k_r-1} - \left(\frac{C_4}{1-k_r} + \frac{C_3}{f+k_r}\left[\frac{r_e}{r}\right]^{f+1}\right) \quad (26)$$

The boundary between the zones of elastic and plastic behavior $r=r_e$, can then be found from the conditions that the radial stress, σ_r, shall be continuous over the boundary and the failure criterion for the peak values are fulfilled.

The ratio r_e/r_i or the extent of the yielding can be solved by trial-and –error form:

$$\frac{r_e}{r_i}=\left[\frac{\sigma_{re}+\dfrac{C_4}{1-k_r}+\dfrac{C_3}{f+k_r}}{p_i+\dfrac{C_4}{1-k_r}+\dfrac{C_3}{f+k_r}\left[\dfrac{r_e}{r}\right]^{f+1}}\right]^{\frac{1}{k_r-1}} \qquad (27)$$

where

$$C_3=\frac{k_r}{S_l S_c}\cdot\frac{2Af}{f+1}E_s Y$$

$$C_4=\frac{k_r}{S_l S_c}\cdot\frac{2Af}{f+1}E_s Y-k_r a_r+a_r$$

The bolt load can then be obtained from the following equation:

$$T=E_s Y\frac{2fA}{f+1}\left[1-\left(\frac{r_e}{r}\right)^{f+1}\right] \qquad (28)$$

where $r\le r_e$.

The bolt load will be in tension with its maximum tensile load at the opening surface and the tensile load will decrease into the rock. This occurrence have been previously proved by Freeman 1978,Zhen Yu *et all.* 1983, and Xueyi 1983. Besides, Indraratna 1990 pointed out that by shear stress transmission from the rock to the bolt surface the axial tension would occur.

- **Plastic bolt in a plastic rock mass**

If the interaction between the bolts an d the rock mass is ideal and the end plate stiff the bolts will be subjected to high loads close to the opening surface. This implies yielding of the bolt if the ultimate strength is exceeded. Under these conditions two zones will be developed around the opening (see Figure6):

- *Plastic rock and elastic bolt,*

- *Plastic rock and plastic bolt.*

In the zone closet to the opening, the stress situation can be derived like the previous solution with the exception that the bolt load has to be replaced by ultimate strength of bolt (yielding load of bolt) $T_{max.}$

As brief, the radial stress can be solved in terms of T_{max}:

$$\sigma_r = \left(p_i + a_r - \frac{T_{\max}}{S_c S_l} \frac{k_r}{k_r - 1} \right) \left[\frac{r}{r_i} \right]^{k_r - 1} - \left(a_r - \frac{T_{\max}}{S_c S_l} \frac{k_r}{k_r - 1} \right) \quad (29)$$

The extension of the zone with plastic rock and plastic bolt (this zone was named with Equivalent Plastic Zone EPZ by Indraratna) can be calculated through the conditions that the bolt load and radial stress shall be continuous over the boundary $r=r_p$. In the zone with plastic rock and elastic bolt, equations (26-28) are valid if r_i is replaced by r_p and p_i with σ_{rp}. The values of r_e and r_p can then be calculated by trial and error with the following equations:

$$T_{\max} = E_s Y \frac{2fA}{f+1} \left[1 - \left(\frac{r_e}{r_p} \right)^{f+1} \right] \quad (30)$$

$$\sigma_{rp} = \left(p_i + a_r - \frac{T_{\max}}{S_c S_l} \frac{k_r}{k_r - 1} \right) \left[\frac{r_p}{r_i} \right]^{k_r - 1} - \left(a_r - \frac{T_{\max}}{S_c S_l} \frac{k_r}{k_r - 1} \right) \quad (31)$$

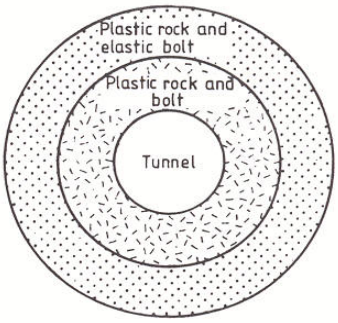


Figure 6. Rock and bolt conditions in the rock mass

- **Plastic deformation of the grouting material**

Common types of grouted bolt are not equipped with a special anchor head. For this types of bolt the bolt load will be zero at the opening face and increase inwards. The plastic deformations of the rock mass will be largest close to the opening surface and the shear strength of the grouting may be exceeded, in which case sliding will occur. The bond between the bolt and the rock mass is considered to be plastic and only transfer a constant shear load per unit length, τ, to the bolt. If it is then assumed that the shear load can be equally distributed out into the rock mass, the following equations are obtained:

$$\sigma_t - \sigma_r - r\frac{\partial \sigma_r}{\partial r} = -\frac{\tau r_i}{S_c S_l} \qquad (32)$$

$$k_r = \frac{\sigma_t + a_r}{\sigma_b + a_r} \qquad (18)$$

$$T = -\tau(r - r_i) \qquad (33)$$

The load in the bolt will increase linearly from zero value at the opening face. the shear load, τ, transferred from the bolt can be interpreted as a volume load radial directed out from the opening. The stresses acting on the rock, σ_b, will not be influenced by the bolt load,T, but indirectly by the shear load. The only connection between the rock and the bolt is the shear load and not the total bolt load. The radial stress, σ_r, wills for this case be the same as the stresses acting on the rock mass, σ_b. The partial differential equation of equilibrium will be different from the earlier case since the shear load will influence the equilibrium of a infinitesimal element as shown in Figure 7. Under these conditions, two zones will be developed surrounding the opening as shown in Figure 8. By combining equations 32 and 18, a distinguished differential equation will be established and using the boundary conditions at opening face the radial stress will then be:

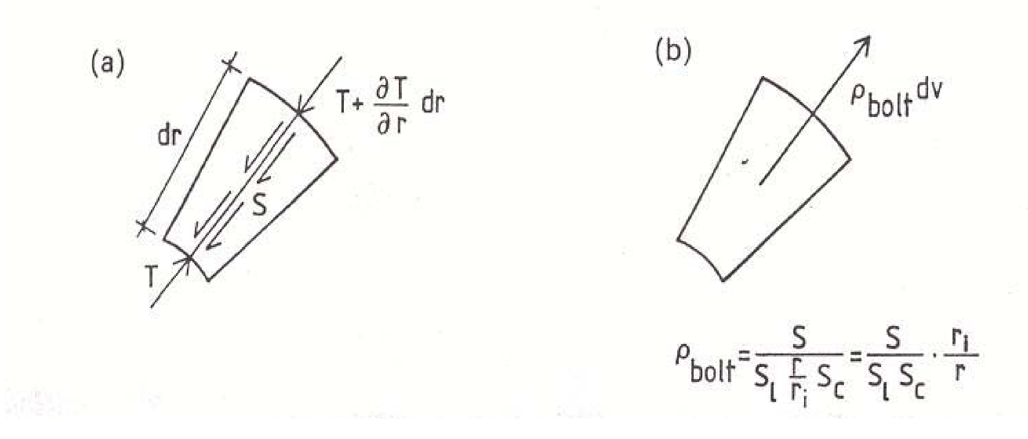


Figure 7. Stresses in the bolt and rock at plastic deformation in the grouting material (a) Stresses in the bolt. (b) Stresss in the rock.

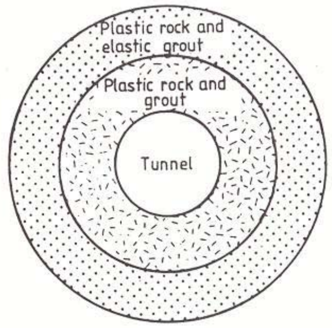


Figure 8. Rock and grout conditions in the rock mass

$$\sigma_r = \left(p_i + a_r - \frac{\tau r_i}{S_c S_l} \frac{1}{k_r - 1} \right) \left[\frac{r}{r_i} \right]^{k_r - 1} - \left(a_r - \frac{\tau r_i}{S_c S_l} \frac{1}{k_r - 1} \right) \quad (34)$$

The extension of the zone with plastic rock and plastic bolt (this zone was named with Equivalent Plastic Zone EPZ by Indraratna) can be calculated through the conditions that the bolt load and radial stress shall be continuous over the boundary $r=r_p$. In the zone with plastic rock and elastic bolt, equations (26-28) are valid if r_i is replaced by r_p and p_i with σ_{rp}. The values of r_e and r_p can then be calculated by trial and error with the following equations:

$$T_p = -\tau (r_p - r_i) \quad (35)$$

$$T_p = E_s Y \frac{2fA}{f+1} \left[1 - \left(\frac{r_e}{r_p} \right)^{f+1} \right] \quad (36)$$

$$\sigma_{rp} = \left(p_i + a_r - \frac{\tau r_i}{S_c S_l} \frac{1}{k_r - 1} \right) \left[\frac{r_p}{r_i} \right]^{k_r - 1} - \left(a_r - \frac{\tau r_i}{S_c S_l} \frac{1}{k_r - 1} \right) \quad (37)$$

- **Elastic bolt with nut and end-plate and plastic deformation of the grout material**

For a grouted rock-bolt with a nut and end plate, plastic deformation can occur in the grouting material before the yield strength of the bolt reached, due to low bond strength. Sliding in the bond between rock and the bolt then starts at the opening surface where the grouting material is assumed to be in a plastic condition and only able to transfer a constant shear load

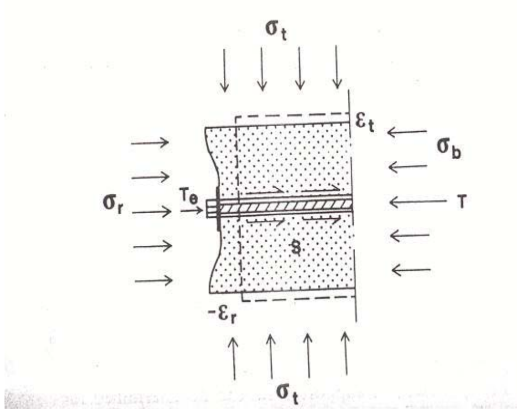


Figure 9. Interaction between rock mass and bolt with end plate. Local deformations of the rock mass under the end plate is shown.

between rock mass and bolt. It is interesting at this point that the bolt load is not zero at the opening surface because of the interaction with the end-plate. The magnitude of the bolt load at the opening surface depends on the bearing capacity of the rock mass under the end plate, see; Figure 9.

Similar to previous model, two zones with different conditions in the support and the rock will develop (see Figure 8). According to the theory of elasticity and assuming a circularly distributed load, the deformation of the rock mass under the end plate is obtained by:

$$u_e = \frac{T_e(1-\nu^2)}{ED} \quad (38)$$

where

T_e= bolt load due to effect of end-plate

E= Young's modulus of the rock mass

D= diameter of the end-plate

The deformation in the rock mass under the end plate is also governed by:

$$u_e = (u_i - u_p) - (u_{gr} + u_s), \quad (39)$$

where

u_i= deformation of rock mass at surface, $r=r_i$,

u_p= deformation of rock mass at $r=r_p$,

u_{gr}= deformation of bolt due to bond load,

u_s= deformation of bolt due to loading of end-plate.

Rock deformation at the opening surface, *ui*, and at the boundary between plastic and elastic grout, u_p, is calculated from:

$$u_r = -\frac{r\,A}{f+1}\left[2\left(\frac{r_e}{r_i}\right)^{f+1} + (f-1)\right] + r_i A \quad (40)$$

where, $r=r_i$ and $r=r_p$ respectively and A and f are computed by equations (8) and (9). The deformation in the bolt is:

$$u_{gr} = -\frac{\tau x^2}{2YE_S} \quad (41)$$

$$u_s = \frac{T_e x}{YE_s} \quad (42)$$

where $x=r_p-r_i$

The equilibrium differential equation will be:

$$\sigma_t - \sigma_r - r\frac{\partial \sigma_r}{\partial r} = -\frac{\tau r_i}{S_c S_l} \quad (32)$$

$$k_r = \frac{\sigma_t + a_r}{\sigma_b + a_r} \quad (18)$$

$$T = -\tau(r - r_i) + T_e \quad (43)$$

The solution of above differential equation is like previous case, hence:

$$T_p = -\tau(r - r_i) + T_e \quad (44)$$

$$T_p = E_s Y \frac{2fA}{f+1}\left[1 - \left(\frac{r_e}{r_p}\right)^{f+1}\right] \quad (45)$$

$$\sigma_{rp} = \left(p_i + a_r - \frac{\tau r_i}{S_c S_l}\frac{1}{k_r - 1}\right)\left[\frac{r_p}{r_i}\right]^{k_r - 1} - \left(a_r - \frac{\tau r_i}{S_c S_l}\frac{1}{k_r - 1}\right) \quad (46)$$

Where T_e is obtained from equations 38 to 42 as:

$$T_e = \left(\frac{(u_i - u_p) + \frac{\tau x^2}{2YE_s}}{\left[\frac{(1-\nu^2)}{ED} + \frac{x}{YE_s}\right]}\right) \quad (47)$$

A hypothetical example is shown to elucidate the different performances of grouted bolt in the aforementioned analyses. Figure 10 shows the analytical bolt load distribution curves along the bolt according to equations 28,30,35, and 43. The analytic solutions correspond to:

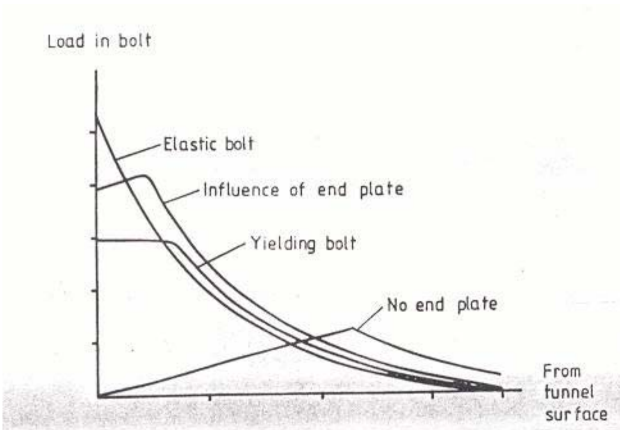


Figure 10. The different analytic bolt load distributions of a grouted bolt.

i. The bolt is in elastic condition and the interaction between rock mass and bolt is ideal. The required bond strength, highest close to the opening surface and decreasing inwards, is numerically equal to the inclination per unit length of the bolt load curve in every point along the bolt. The stiffness of the end plate is infinite.

ii. In the outer part of the bolt, the yield strength of the bolt is exceeded. The interaction between rock mass and bolt is ideal and a perfect plastic behavior of the bolt is assumed.

iii. The bolt is without end-plate and in elastic condition. The shear strength of the grouting material is exceeded in the outer part of the bolt. In that zone, the bolt load only depends on the residual capacity of the grout to transfer load between rock mass and bolt.

iv. The bolt is in elastic condition. The shear strength of the grouting material is exceeded in the outer part of the bolt and the load depends on the local deformations of the rock mass under the end plate and the residual capacity of the grout to transfer load between rock mass and bolt. The end-plate acts theoretically as a circular spread load.

Grouted bolts themselves are not considered to establish any radial support pressure, P_i, on the rock surface, so equilibrium for the ground reaction curve is reached as for unsupported rock when $P_i=0$. The principal effect of grouted bolts, compared to the unsupported roc mass, is that the stability of rock mass is improved as the bolts through tension load influence the strength of the rock mass and the volume expansion at failure. In the previous studied conducted by Hoek and Brown (1980), these effect of grouted bolts were mentioned, but no analytical solution was presented.

This analytical solution discussed here, was employed at the Kielder experimental opening to be verified. Based on results obtained from both this solution and field measurement, it was concluded that there was a good consistency between results(Freeman,1978,Ward *et al.* 1983)

5.2.INDRARATNA & KAISER CONSTITUTIVE MODEL

Indraratna (1987,1988,1989,1990) developed an analytical model which represents the behavior of a reinforced rock mass near a circular underground opening in a homogeneous, isotropic and uniform field stress. The theory adopts the concepts of elasto-plasticity and considers a proper interaction mechanism between the ground and the grouted (friction) bolts to accommodate for the influence of bolt/ground interaction, size of opening and the bolt pattern on yielding and opening wall displacement. It highlights the influence of bolt pattern on the extent of the yield zone and opening deformation in terms of convergence reduction. In this model, two appealing dimensionless parameters namely bolt density and normalized convergence ratio were defined. It is also so interesting that a new parameter, neutral point of bolt, developed by Xueyi (1983) was incorporated into analysis. Such dimensionless parameters were considered to incorporate the opening convergence into the bolt pattern for a given bolt length. Furthermore, one of the important tasks conducted by Indraaratna (1990) was that he incorporated this solution into empirical design approaches (rock mass classification systems) especially Bieniawski's RMR system and acceptable agreement was resulted. This constitutive model deals with mainly the effect of bolts on the stress and displacement field near an opening. The elasto-plastic model presented in the following constitutes an extension of the approach introduced earlier by Kaiser et all.(1985) for the purpose of assessing the influence of fully grouted rock-bolts on opening behavior.

5.2.1. Elasto-plastic behavior of model

The assumption of homogeneity, isotropy and linear elasticity prior to yielding of the rock mass are made to simplify the mathematical treatment. In addition, the existence of a uniform field stress can often be justified for deep excavations. Yield initiation is assumed to occur following a linear Mohr-Coulomb failure criterion. In particular, near a opening where the confining(radial)stress is a minimum, the fracture initiated by a relatively large deviator stress(σ_θ-σ_r) can be modeled adequately by a linear failure envelope. Owing to the fact that some "strain-weaking behavior" is observed in most rocks, this is simulated by an elastic, brittle-plastic model which is characterized by an instantaneous strength drop at peak .The yield envelope is considered to be linear although the post peak strength is reduced. The principal stresses in the plastic or yield zone can be described by:

$$\sigma_\theta = m\sigma_r + s\sigma_c \quad (18b)$$

where

$$m = \tan^2\left(45 + \frac{\phi}{2}\right) \qquad 0<s<1$$

The parameter s is a measure of the degree of strength loss occurring immediately after the peak strength is reached. In uniaxial compression, s is almost zero, whereas it approaches unity if the perfectly elasto-plastic state is attained in treaxial compression.

The strains in the plastic zone are the sum of elastic and plastic components. The elastic component in the plastic zone has been determined by assuming identical constants to those of the elastic rock (E,ν)and by applying Hooke's laws. The plastic strains are governed by an appropriate flow rule postulated for a yielding behavior. Since the extent of yielding is dependent on the dilation characteristics of the yielded material, the flow rule must accommodate the influence of dilation. A non associated flow rule for this analysis is taken into account as discussed in the part 3 .

5.2.2 Neutral point of rock-bolt

The shear stress distribution (τ_z) along a grouted bolt can be represented by (Xueyi, 1983):

$$-dQ_z = \pi d \tau_z dz \quad \text{(48a)}$$

$$\tau_z = -\frac{1}{\pi d}\cdot\frac{dQ_z}{dz} = -\frac{r_b}{2}\cdot\frac{d\sigma_z}{d_z} \quad \text{(48b)}$$

Where bolt diameter $d=2 \times$ *bolt radius* (r_b), Q_z is the axial load distribution and σ_z is the axial stress distribution along the bolt.

The shear stress is related to the first derivative of the axial stress; hence, a zero value of τ_z defined as the neutral point must exist where the axial stress attains a maximum. A model for stress distribution associated with grouted bolts has been proposed initially by Freeman, 1978 based on field measurements from the Kielder experimental opening, and later by Xueyi 1983, also based on field observations. This model, illustrated diagrammatically in Figure 11,clearly demonstrates the occurrence of the neutral point the location of the maximum axial stress. It further exhibits points of inflection on the axial stress distribution associated with the

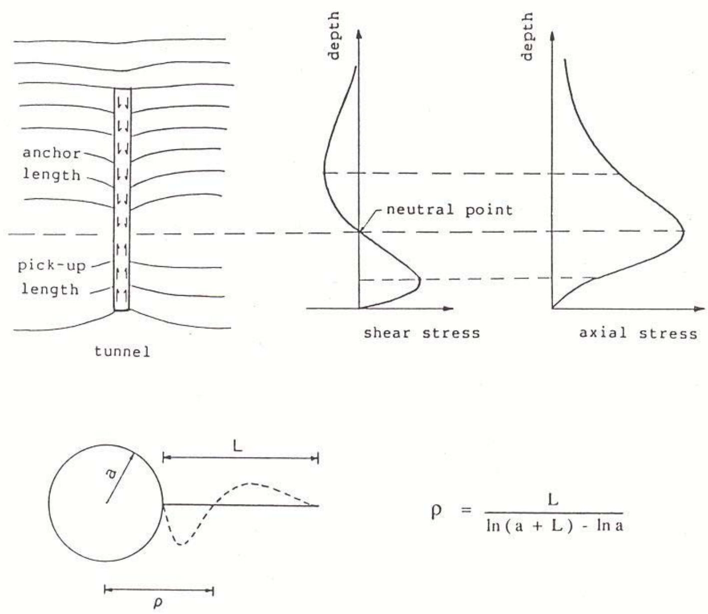


Figure 11. Stress distribution model for grouted bolts (*after* Xueyi, 1983).

maximum and minimum of the shear stress distribution, where:

$$\frac{d\tau_z}{dz} = \frac{d^2\sigma_z}{dz^2} = 0 \quad (49)$$

The shear stress distribution is characterized by the division of the bolt into a pick-up length and an anchor length, on either side of the neutral point. This is justified mathematically by considering the equilibrium of the grouted bolt relative the surrounding rock. The pick-up length restrains the ground displacements towards the opening whereas the anchor length is restrained by the rock. The equilibrium of the bolt relative to the rock is thereby ensured as a result of the shear stresses acting in opposite directions along the pick-up length and anchor length, respectively. The relative displacement at the neutral point is essentially zero. Yu and Xian (1983) have independently investigated the interaction mechanisms of the fully grouted bolts and have provided further theoretical support for the above described model. The location of the neutral point along the bolt has been determined by equilibrium considerations, and it is given by:

$$\rho = \frac{L}{\ln\left[1 + \left(\frac{L}{a}\right)\right]} \quad (50)$$

where L is bolt length and a is opening radius. According to observations, it was seen that $\rho \sim 0.45L + a$ and $L \sim (20\text{-}30)\ d$

For a axisymmetrical problem and considering identical bolt with equal spacing along the opening axis and around the circumference, the tangential bolt spacing around the opening is defined by the product of the opening radius and the angle between two adjacent bolts(i.e. $S_T = a\theta$) see Fig12.

5.2.3. General elastic, brittle-plastic model around circular opening

This part of study intended to introduced an analytical of circular opening in isotropic, elastic- brittle plastic continuum undertaken by Kaiser et al.(1985). Since in the following analysis Kaiser's solution will be a base, it is, hence, necessary to acquaint with that solution.

a) Failure Criterion

The linear Mohr-Coulomb criterion is applied with a reduction in post-peak strength, as given by the following relationships:

$$\sigma_\theta = m\sigma_r + s\sigma_c \quad (18b)$$

$0<s<1$

The parameter s a measure of the degree of strength loss occurring instantaneously after the peak (failure) stress.

b) Stress in the yielded zone

The combination of the equilibrium equation and failure criterion results in the following ordinary differential equation:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (51)$$

$$\sigma_\theta = m\sigma_r + s\sigma_c$$

results in

$$\frac{d\sigma_r}{dr} + \frac{(1-m)\sigma_r}{r} = \frac{\sigma_{cr}}{r} \quad (52)$$

where

$$m = \tan^2\left(45 + \frac{\phi}{2}\right) = \frac{1+\sin\phi}{1-\sin\phi} \qquad \text{and} \quad \sigma_{cr} = s.\sigma_c$$

The shear stress ($\tau_{r\theta}$) at any given radial distance is zero for axisymmetric deformation under plane strain condition. For an unsupported opening where bolt density parameter ($\beta=0$), the solutions of Equ.52 are given by:

$$\sigma_r = \left[\frac{s\cdot\sigma_c}{m-1}\right]\left[\left(\frac{r}{a}\right)^{m-1} - 1\right] \quad (53)$$

$$\sigma_\theta = \left[\frac{s\cdot\sigma_c}{m-1}\right]\left[m\left(\frac{r}{a}\right)^{m-1} - 1\right] \quad (54)$$

the above solution is the same for both the geomechanical model and the actual excavation.

c) Stresses in the outer elastic zone

The stress distribution in the elastic zone is equivalent to that of a larger opening of radius R, supported by an uniform internal stress σ_{ri} under the same external field stress. R is the radial distance to the outer limit of the yielding zone surrounding the opening.

At the elastic plastic boundary ($r=R$), the internal stresses are given by:

$$\sigma_{ri} = \left[\frac{s\cdot\sigma_c}{m-1}\right]\left[\left(\frac{R}{a}\right)^{m-1} - 1\right] \quad (55)$$

$$\sigma_{\theta i} = \left[\frac{s\cdot\sigma_c}{m-1}\right]\left[m\left(\frac{R}{a}\right)^{m-1} - 1\right] \quad (56)$$

In the elastic zone, the stress distributions are given by:

$$\sigma_r = \sigma_o\left[1-\left(\frac{R}{r}\right)^2\right] + \sigma_{ri}\left(\frac{R}{r}\right)^2 \quad (57)$$

$$\sigma_r = \sigma_o\left[1+\left(\frac{R}{r}\right)^2\right] - \sigma_{ri}\left(\frac{R}{r}\right)^2 \quad (58)$$

where

$$\sigma_r + \sigma_\theta = 2\sigma_o \quad (59)$$

d) Radius of the yielded zone

The plastic zone radius R can be determined by assuming continuity of radial stress at the elastic-plastic boundary. It is also assumed that the field boundaries are far enough from the opening, such that their influence on the solution on the solution for R is negligible.

Equating the expressions for σ_{ri} at $r=R$, obtained for the elastic and plastic zones respectively, the normalized plastic zone radius (R/a) can be derived as follows:

$$\frac{R}{a} = \left[1+\frac{1}{s}\left(\frac{m-1}{m+1}\right)\left(\frac{2\sigma_0}{\sigma_c} - 1\right)\right]^{1/(m-1)} \quad (60)$$

e) Strains in the elastic zone

Hook's laws can be applied to determine the radial and tangential strains in the elastic region surrounding the plastic zone.

$$\varepsilon_r = \frac{1-\nu^2}{E}\left[\sigma_r - \left(\frac{\nu}{1-\nu}\right)\sigma_\theta\right] \quad (61)$$

$$\varepsilon_\theta = \frac{1-\nu^2}{E}\left[\sigma_r - \left(\frac{\nu}{1-\nu}\right)\sigma_r\right] \quad (62)$$

substitution of the expressions for stresses (Equations 57, 58) in the above relationships provides the strain fields for the model test under plane strain conditions ($\gamma_{r\theta}$):

$$\varepsilon_r = \frac{\sigma_o}{2G}(1-2\nu) + \frac{\sigma_o}{2G}\left[1-\left(\frac{\sigma_{ri}}{\sigma_o}\right)\right]\left(\frac{R}{r}\right)^2 \quad (63)$$

$$\varepsilon_\theta = \frac{\sigma_o}{2G}(1-2\nu) - \frac{\sigma_o}{2G}\left[1-\left(\frac{\sigma_{ri}}{\sigma_o}\right)\right]\left(\frac{R}{r}\right)^2 \quad (64)$$

where σ_{ri} is the radial stress at $r=R$. The term $\sigma_o(1\text{-}2v)/2G$ is the initial elastic deformation of the plate without the opening. The other term is the deformation due to excavation. The deformation of the laboratory model is the combination of both terms.

f) Strains in the plastic zone

The total strains in the plastic zone are made up of both elastic and plastic strains ($\varepsilon^t=\varepsilon^e+\varepsilon^p$). Hook's law has been applied to calculate the elastic strains, which are given by the following expressions:

$$\varepsilon_\theta{}^e = \frac{1}{2G}\left[\frac{s\cdot\sigma_c}{m-1}\right]\left[(m-m\nu-\nu)(\frac{r}{a})^{m-1}+(2\nu-1)\right] \quad (65)$$

$$\varepsilon_r{}^e = \frac{1}{2G}\left[\frac{s\cdot\sigma_c}{m-1}\right]\left[(1-\nu-m\nu)(\frac{r}{a})^{m-1}+(2\nu-1)\right] \quad (66)$$

The continuity of total strains across the elastic-plastic boundary requires a specific tangential plastic strain associated with strength loss after peak to occur immediately. The magnitude of this plastic strain at $r=R$ is given by:

$$\varepsilon_\theta{}^R = \frac{\sigma_c}{2G}(1-\nu)(1-s) \quad (67)$$

Clearly, the plastic strains become zero at the elastic-plastic for perfectly plastic material with $s=1$.

Substitution of Equations 65 and 66 and the flow rule (Equ.13) in the total strain compatibility condition $\frac{d\varepsilon_\theta{}^t}{dr}+(\varepsilon_\theta{}^t-\varepsilon_r{}^t)/r=0$ provides the following differential equation:

$$\frac{d\varepsilon_\theta{}^p}{dr}+\varepsilon_\theta{}^p(1+\alpha) = -\left[\frac{s\sigma_c}{2G}\right](1-\nu)(m+1)(\frac{r}{a})^{m-1} \quad (68)$$

The solution of the above equation with the boundary condition stated in Equation 67 is given by:

$$\varepsilon_\theta{}^p = s\sigma_c\left[\frac{1-\nu}{2G}\right]\left[\frac{m+1}{m+a}\right]\left[\left(\frac{R}{r}\right)^{m+\alpha}-1\right]\left(\frac{r}{a}\right)^{m-1}+\varepsilon_\theta{}^R\cdot\left(\frac{R}{r}\right)^{1+\alpha} \quad (69)$$

and

$$\varepsilon_r{}^p = -\alpha\cdot\varepsilon_\theta{}^p \quad (70)$$

The addition of the corresponding Equations 65,66 and 69,70 gives the total strains in the plastic zone for the boundary conditions of the model test, where:

$$\varepsilon_\theta^t = \varepsilon_\theta^e + \varepsilon_\theta^p \qquad and \quad \varepsilon_r^t = \varepsilon_r^e + \varepsilon_r^p \qquad (71)$$

g) Radial displacement field

The displacement field can be obtained directly by the following strain-displacement relationships which satisfy the compatibility conditions:

$$\varepsilon_r = \frac{\partial u_r}{\partial r} \quad \text{and} \quad \varepsilon_\theta = \frac{u_r}{r}+\frac{1}{r}\cdot\frac{\partial u_\theta}{\partial\theta} \quad (72)$$

the conditions of plane strain under axisymmetric deformation ($\gamma_{r\theta}$) imply that the total strains are independent of the tangential strain components. Therefore, the radial displacement field can be readily evaluated from any of the following expressions:

$$\frac{u_r}{r} = \varepsilon_\theta{}^t \quad \text{or} \quad u_r = \int \varepsilon_r{}^t\cdot dr \qquad (73)$$

Elasto-plastic opening convergence can be subsequently determined by substituting $r=a$ in the above expressions.

The displacement field is then given by:

$$\frac{u_r}{r} = \left(\frac{\sigma_c}{2G}\right)\left\{\begin{array}{l} s\left(\frac{r}{a}\right)^{m-1}\left(\left[\frac{m(1-\nu)-\nu}{m-1}\right]+(1-\nu)\left[\frac{m+1}{m+\alpha}\right]\left[\left(\frac{R}{r}\right)^{m+\alpha}-1\right]\right)+ \\ \frac{s(2\nu-1)}{m-1}+(1-\nu)(1-s)\left(\frac{R}{r}\right)^{\alpha+1} \end{array}\right\} \quad (74)$$

and the opening closure by:

$$\frac{u_a{}^{ep}}{a} = \left[\frac{\sigma_c(1-\nu)}{2G}\right]\left[s\left\{1+\left[\frac{m+1}{m+\alpha}\right]\left[\left(\frac{R}{a}\right)^{m+\alpha}-1\right]\right\}+(1-s)\left(\frac{R}{a}\right)^{\alpha-1}\right] \quad (75)$$

the closure of a opening under external load application in a linear elastic material is:

$$\frac{u_a{}^e}{a} = \sigma_o(1-\nu)/G \qquad (76)$$

The total opening convergence normalized to the elastic convergence is then:

$$\frac{u^{ep}}{u^e} = \left(\frac{\sigma_c}{2\sigma_o}\right)\left[s\left\{1+\left[\frac{m+1}{m+\alpha}\right]\left[\left(\frac{R}{a}\right)^{m+\alpha}-1\right]\right\}+(1-s)\left(\frac{R}{a}\right)^{\alpha-1}\right] \qquad (77)$$

which is independent of the deformation properties of the elastic material.

5.2.4.Influence of bolting on strength parameters

The equilibrium of an element near an unsupported opening in accordance with theory of elasticity (Figure 12) can be represented by:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (51)$$

Combination of the linear Mohr-Coulomb linear failure criterion, the above equilibrium equation leads to:

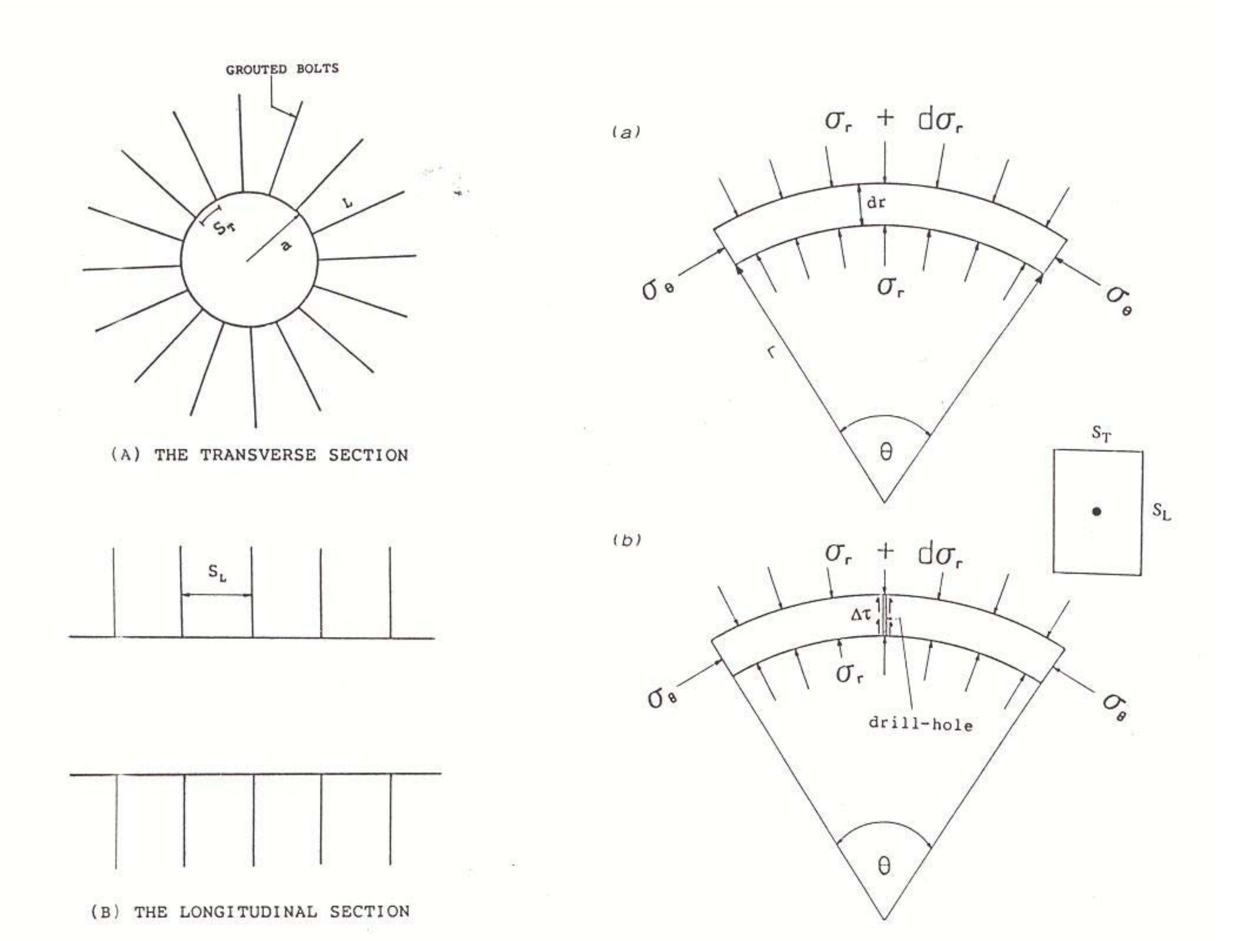


Figure 12. Equiblirium consideration for bolt-rock mass intearction

$$\frac{d\sigma_r}{dr} + \frac{(1-m)\sigma_r}{r} = \frac{\sigma_{cr}}{r} \quad (52)$$

on the other hand, in a bolted element (Figure12), the equilibrium condition for this segment of longitudinal length S_L can be represented by the following equation, if the additional radial force due to shear stresses along the borehole is assumed to be given by

$$\Delta T = \pi d \sigma_\theta \lambda dr \quad (78)$$

$$\frac{d\sigma_r}{dr} + \frac{[1-m(1+\beta)]\sigma_r}{r} = \frac{\sigma_{cr}(1+\beta)}{r} \quad (79)$$

the bolt density parameters is defined by:

$$\beta = \frac{\pi d \lambda}{S_L \theta} = \frac{\pi d \lambda a}{S_L S_T} \quad (80)$$

as we see the bolt density parameter is dimensionless. It reflects the relative density of bolts with respect to the opening perimeter and takes into consideration the sheer stresses on the bolt surface, which oppose the rock mass displacements near the opening wall.

The magnitude of β can be increased by:

1. Decreasing the bolt spacing
2. Increasing the bolt surface area
3. Increasing the roughness of bolt surface

In practice, the value of β varies between 0.05 and 0.20 for most cases. In a few case histories such at the Enasan Opening, analyzed by Indraratna(1987) very high values for β (in excess of 0.4)were reached by very intensive bolting patterns. The friction factor, λ, is analogous to the coefficient of friction. It relates the mean mobilized shaer stress to the stress applied normal to the bolt surface. The magnitude of λ for smooth rebars falls in the range tan (φ_g/2)< λ<tan (2φ_g/3) and for shaped rebars approaches tanφ_g, depending on the degree of adhesion (bond strength) at the bolt/grout interface. The friction angle of a hardened grout (cement or resin) is comparable to that of most intact rock. The ratios β/λ for many case histories determined by Indraratna(1990a) indicate that the β/λ varies between 0.12 and 0.41 in a normal manner.

The bolt length, another important parameter for controlling displacements, is not included in the bolt density parameter because the effect of a bolt depends on its length relative to the radius of the yield zone. The shear stress distribution and, hence, the location of the neutral point are directly related to the bolt length, the extent of the plastic zone and the strength reduction in this zone. As will be shown later, the extent of the yield zone and the opening wall displacements can be effectively reduced by increasing the bolt length.

5.2.5. Concept of equivalent strength parameters

Equations 52 and 79 describe the equilibrium condition of the unsupported and reinforced segments, respectively. Both equations contain the same algebraic structure if the terms $m\ (1+\beta)$ and $\sigma_{cr}(1+\beta)$ are replaced by the equivalent parameters m^* and $\sigma_{cr}{}^*$,respectively. Equation 79 for the bolted composite can then be simplified to:

$$\frac{d\sigma_r}{dr} + \frac{(1-m^*)\sigma_r}{r} = \frac{\sigma_{cr}{}^*}{r} \quad (81)$$

Where

$$m^* = m(1+\beta) \quad \text{and} \quad \sigma_{cr}{}^* = \sigma_{cr}(1+\beta)$$

grouted bolts create a zone of improved, reinforced rock in the region defined by the pick-up length of the bolts. Within this zone, the friction angle and the uniaxial compressive strength of the rock mass are increased. Therefore the degree of stabilization around the opening wall is a function of the bolt density parameter, β. Hoek and Brown (1980) had recognized the increase in apparent strength parameters due to fully grouted bolts but have not presented a theoretical model.

It should be noted that unlike equivalent friction angle φ^*, no expression defining the equivalent cohesion was considered. Some criticizes arose for this constitutive model due to that deficiency (Oreste and Peila, 1996).

5.2.6.Concept of equivalent plastic zone

Grouted bolts have the effect of improving the weakened or loosened zone by increasing the apparent strength. The extent of the plastic zone is directly related to this rock mass properties and any improvement of the rock strength must reduce the extent of the zone of overstressed rock, if the bolts are installed soon after excavation close to the face. Consequently, the plastic zone of a bolted opening is smaller than that of an unsupported opening in the same ground. This is termed the " **Equivalent Plastic Zone**" because it is the yield zone in a material of improved properties simulating a behavior equivalent to the bolted rock mass. A reduction of the apparent plastic zone in turn curtails opening wall displacement. The extent of the plastic zone is influenced by the material parameters φ and σ_c and is independent of the elastic parameters E and ν. Indraratna's solution does not consider an increased stiffness of the reinforced rock because the elastic component of strain or displacement is assumed to be small in comparison to the plastic component. The following factors directly affect the radius R^* of the equivalent plastic zone:

- Bolt density parameter,β
- Bolt length,L
- Radius of the neutral point of the bolt,ρ
- Opening radius,a
- Fiels stress,P_o

It should be noted that the above parameter are only with respect of rock-bolt. It is further interesting to say that four of five parameter were also recognized by empirical approach under taken by Ünal(1983).

In comparison with Ünal's rock load height concept, it can be deduced here that rock load height is similar to plastic (yielding) zone. Opening span, the effect of field stress particularly horizontal stress, bolt pattern are considered as the main parameters affecting the failure (overstressed) zone of an opening in both approaches. Another convincing argument arising here is the role of rock mass classification systems such as MRMR. Why Indraratna does not consider the one of the rock mass classification systems is that the plastic zone and equivalent plastic zone are calculated based on analytical expression (theory of elasticity) whereas Ünal's empirical solution takes into account the rock mass classification ratings.

The determination of the equivalent plastic zone EPZ radius, R^*, must be divided into three categories depending on the location of the interface between the elastic rock and the equivalent plastic zone relative to the neutral point and the bolt length (Figure13):

I. $R^*<\rho<(a+L)$ minimal yielding

II. $\rho<R^*<(a+L)$ major yielding

III. $R^*>(a+L)$ excessive yielding

Solutions of the Equivalent Plastic Zone Radius

Category (I) $R^*<\rho<(a+L)$ **minimal yielding**

The condition of minimal yielding occurs either at relatively low field stress or when the bolts are excessively long. in these situations, the extent of the plastic zone is confined within the pick-up length of the bolt and four distinct zone can be identified by the location of the plastic zone relative to the neutral point and the bolt ends.

A. Zone 1: $a<r<R^*$ r is point of interest

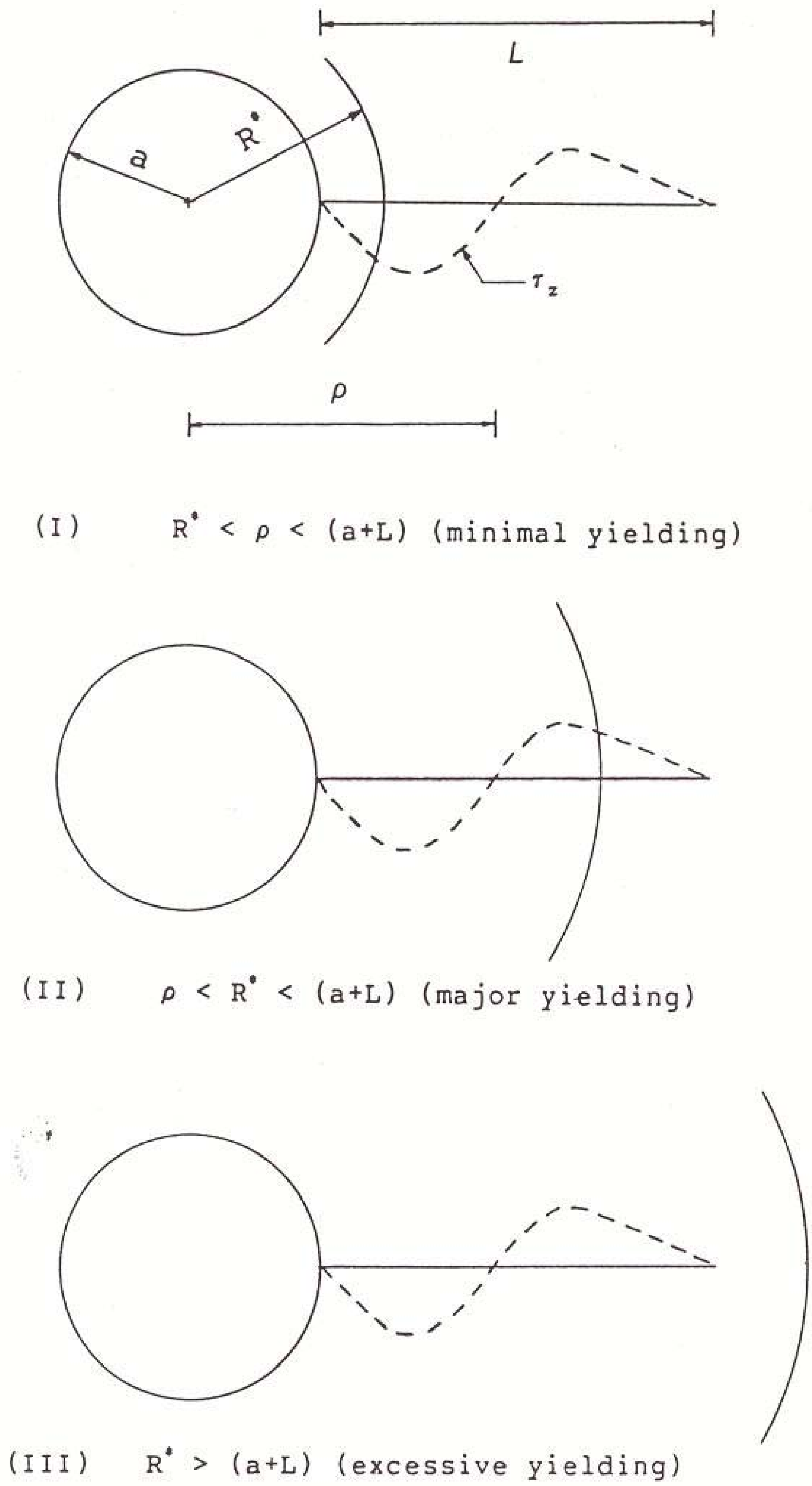


Figure 13. Categorazation of the extent of yielding (*after* Intraratna, 1987)

In this region of the pick-up length, the ground displacements toward the opening are resisted by positive shear stress. The equivalent stress field in this zone is represented by:

$$\sigma_r = \left(\frac{s\sigma_c{}^*}{m^* - 1}\right)\left[\left(\frac{r}{a}\right)^{m^*-1} - 1\right] \quad (82)$$

$$\sigma_\theta = m^*\sigma_r + s\sigma_c{}^* \quad (83)$$

where

$m^* = m(1+\beta)$ and $\sigma_c{}^* = \sigma_c(1+\beta)$

B. Zone 2: $R^*<r<\rho$

This part of the elastic zone is confined to the pick-up length of the bolt. The elatic stress fields in this zone are given by:

$$\sigma_r = \sigma_o\left[1 - \left(\frac{R^*}{r}\right)^2\right] + \sigma_{rR}\left(\frac{R^*}{r}\right)^2 \quad (84)$$

$$\sigma_\theta = \sigma_o\left[1 - \left(\frac{R^*}{r}\right)^2\right] - \sigma_{rR}\left(\frac{R^*}{r}\right)^2 \quad (8)$$

the peak tangential stress, $\sigma_{\theta R}$, at the elasto-plastic interface for $s=1$ is given by the following condition:

$$\sigma_{\theta R} = m^*\sigma_{rR} + \sigma_c{}^*$$

the radial stress at the elasto-plastic boundary σ_{rR} is derived by substituting $r=R^*$ in the latter equations:

$$\sigma_{rR} = \frac{2\sigma_o - \sigma_c{}^*}{m^* + 1} \quad (86)$$

C. Zone 2: $\rho<r<(a+L)$

This part of the elastic zone is contained within the anchor length of the bolt. The radial and tangential stress fields are given by:

$$\sigma_r = \sigma_o\left[1 - \left(\frac{\rho}{r}\right)^2\right] + \sigma_\rho\left(\frac{\rho}{r}\right)^2 \quad (87)$$

$$\sigma_\theta = \sigma_o\left[1+\left(\frac{\rho}{r}\right)^2\right]-\sigma_\rho\left(\frac{\rho}{r}\right)^2 \quad (88)$$

where

$$\sigma_p = \sigma_o\left[1-\left(\frac{R^*}{\rho}\right)^2\right]+\sigma_{rR}\left(\frac{R^*}{\rho}\right)^2 \quad (89)$$

D. Zone 4: *r > (a+L)*

This outermost elastic region, beyond the bolt, is in virgin rock and the elastic stresses are given by:

$$\sigma_r = \sigma_o\left[1-\left(\frac{a+L}{r}\right)^2\right]+\sigma_L\left(\frac{a+L}{r}\right)^2 \quad (90)$$

$$\sigma_\theta = \sigma_o\left[1+\left(\frac{a+L}{r}\right)^2\right]-\sigma_L\left(\frac{a+L}{r}\right)^2 \quad (91)$$

where

$$\sigma_L = \sigma_o\left[1-\left(\frac{\rho}{a+L}\right)^2\right]+\sigma_\rho\left(\frac{\rho}{a+L}\right)^2 \quad (92)$$

the radial distance to the neutral point is given by Equation 50 ,as discussed earlier. At the elastic-plastic interface, the assumption of continuity of radial stress gives

$$\sigma_{rR} = \frac{2\sigma_o-\sigma_c^*}{m^*+1} = \left(\frac{s\sigma^*{}_c}{m^*-1}\right)\left[\left(\frac{R^*}{a}\right)^{(m^*-1)}-1\right] \quad (93)$$

the solution of above equation provides the normalized radius of the equivalent plastic zone (EPZ):

$$\frac{R^*}{a} = \left[1+\frac{1}{s}\left(\frac{m^*-1}{m^*+1}\right)\left(\frac{2\sigma_o}{\sigma^*{}_c}-1\right)\right]^{1/(m^*-1)} \quad (94)$$

It is obvious that as β tends to zero, the parameters m^* and σ_c^* approach m and σ_c. In other word, above equation becomes identical to that of unsupported case as derived by Kaiser *et al*(1985) i.e.

$$\frac{R}{a} = (\left[\left(\frac{m\ -1}{m\ +1}\right)\right]\left[\frac{(2\sigma_o-\sigma_c)}{s\sigma_c}\right]+1)^{1/(m^*-1)} \quad (95)$$

As discussed above, for the category (I) the solutions are solved, in the same way solutions for the categories (II) and (III) are extractable. The condition of major yielding, $\rho<R^*<(a+L)$, occurs when the extent of the plastic zone has propagated beyond the neutral point. In this situation, the plastic zone itself is divided by the neutral point into two zones. Consequently, only the plastic zone region that falls within the pick-up length of the bolt is effectively stabilized by the positive shear stresses. The equivalent plastic zone radius is then given by:

$$\frac{R^*}{a}=\frac{\rho}{a}\left(\frac{1+B_1'}{1+A_1}\right)^{1/(m'-1)} \quad (96a)$$

where

$$B_1'=\frac{1}{s}\left(\frac{m'-1}{m'+1}\right)\left(\frac{2\sigma_o}{\sigma_c'}-1\right) \quad (96b)$$

$$A_1=\left(\frac{m'-1}{m^*-1}\right)\left(\frac{1+\beta}{1-\beta}\right)\left[\left(\frac{\rho}{a}\right)^{(m^*-1)}-1\right] \quad (96c)$$

$m'=m(1-\beta)$ and $\sigma'_c=\sigma_c(1-\beta)$ (96d)

The condition of excessive yielding, $R^*>(a+L)$, occurs either due to large *in-situ* stress in relatively weak rock or as a result of inadequate bolt length. In this situation, the bolt is completely embedded in the yielded rock and no anchorage is provided from the outer elastic zone. The radius of the equivalent plastic zone is now given by:

$$\frac{R^*}{a}=\left[1+\left(\frac{L}{a}\right)\right]\left(\frac{1+B_1}{1+A_2+A_3}\right)^{1/(m-1)} \quad (97a)$$

$$B_1=\frac{1}{s}\left(\frac{m-1}{m+1}\right)\left(\frac{2\sigma_o}{\sigma_c}-1\right) \quad (97b)$$

$$A_2=\left(\frac{m-1}{m'-1}\right)\left[\left(\frac{a+L}{\rho}\right)^{(m'-1)}-1\right] \quad (97c)$$

$$A_3=(1+\beta)\left(\frac{m-1}{m^*-1}\right)\left(\frac{a+L}{\rho}\right)^{(m'-1)}\left[\left(\frac{\rho}{a}\right)^{(m^*-1)}-1\right] \quad (97d)$$

$$m^*=m(1+\beta)$$

$$m'=m(1-\beta) \quad (97e)$$

The strain and displacement fields are determined by the application of Hooke's laws, strain compatibility, flow rule and strain-displacement relationships as discussed earlier.

5.2.7. Discussion of behavior predicted by the analytical model

Evidence to support the analytical predictions has been obtained by laboratory simulations of a model opening; accordingly, results in much interesting finding as briefly given here(Indraratna 1987):

1. It was concluded that as the bolt density parameter β increases, the radial and tangential stress fields approach those predicted for non-yielding, elastic rock, and the radius of equivalent plastic zone (R^*) approaches the opening radius. Further away from opening, the stress field tends toward the far-field stress.

2. For the strain field, as the bolt density parameter β increases, the radial and tangential strains approach the elastic solution. The pronounced reduction of the total tangential strain(ε_θ^t) inside the overstressed zone at the elastic-plastic boundary indicates strengthening of the yielding material by the bolts.

3. Regarding displacement induced, as the distance from the opening wall increases, the effect of bolting on the radial displacement diminishes rapidly and the far field conditions are approached. It is evident that the maximum decrease in strains and radial displacements occurs at the opening wall. Hence, the opening wall convergence can be considered as the most appropriate parameter for a displacement controlled design approach.

5.2.8. Influence of grouted bolts on opening wall stability

The radial strains and displacements at the opening wall are the most fundamental quantities required to evaluate the stability of a opening. In the field they are not only feasible to measure but are also generally reliable. The radial strain and convergence of the reinforced opening wall can be predicted from the following equations, after the magnitude of R^* has been determined for the respective categories I to III:

$$\varepsilon_a{}^* = -\frac{\alpha(1-\nu)}{2G}\left\{\sigma_{cr}M\left[\left(\frac{R^*}{a}\right)^{(m+a)}-1\right]+\sigma_c(1-s)\left[\left(\frac{R^*}{a}\right)^{(1+a)}\right]\right\}-\frac{\nu\sigma_{cr}}{2G} \quad (98)$$

$$\frac{u_a^*}{a} = \frac{(1-\nu)}{2G}\left\{\sigma_{cr}\left[1+M\left[\left(\frac{R^*}{a}\right)^{(m+\alpha)}-1\right]\right]+\sigma_c(1-s)\left[\left(\frac{R^*}{a}\right)^{(1+\alpha)}\right]\right\} \quad (99)$$

where

$$G = \frac{E}{1(1+\nu)}$$

and

$$M = \frac{2}{1+\alpha-\sin\phi(\alpha-1)} \quad (100)$$

the derivation of the above expressions is based on the assumption that both radial displacement and radial stresses are continuous across the elasto-plastic boundaries, regardless of the field stress magnitude. In addition, the post peak parameters α and s are assumed to be constant irrespective of the bolt density; whereas, the elastic parameters G and ν are considered to be characteristic of the original intact material prior to yielding.

5.2.9. Use of displacement control approach for design

The dimensionless ratios R^*/a and u^*/a are both directly dependent on the bolt density parameter β and normalized bolt length (L/a). If β is kept constant for a smaller opening excavated in the same homogeneous and isotropic rock., the ratio u^*/a is not affected if the bolt length is also reduced proportionately(i.e. scaled reduction). However, if the bolt length remains unchanged for a smaller opening radius, the quantity u^*/a decreases for the same β. In contrast, for a larger opening the bolt length must be increased accordingly in order to maintain the same u^*/a ratio for a given β.

The above predictions may not be accurate for a opening excavated in a predominantly jointed medium. This is because; a larger opening intersects a greater number of discontinuities, thereby adopting a behavior equivalent to that of an excavation in a weaker medium.

The applicability of this philosophy was examined by Indraratna 1987 in details. By knowing the properties of rock mass around the opening and post peak parameters of rock α and s, one is able to calculate the extent of the yielding zone and predicted convergence of the unsupported opening as discussed earlier. The next step is to use a bolt pattern with respect to bolt density parameter. Since the bolt length isn't included into bolt density parameter, we can assess the stability based on either bolt density effect or bolt length effect. No matter which treatment is applied, the extent of yielding zone as well as total opening wall convergence decrease.

5.2.10. Normalized convergence ratio

The convergence of a reinforced opening can be presented by the dimensionless ratio u_a^*/u_a, where u_a^* and u_a are the total convergence of the reinforced and unsupported opening respectively at the same stress level. The total opening convergence includes both the elastic and plastic displacements. For a given field stress, u_a^* is less than u_a but it approaches u_a when the bolt density (β) or the bolt length (L) tends to zero.

The normalized convergence ratio decreases as the intensity of bolting increases. It obtains a minimum value when u_a^* tends to u_e, the elastic portion of the total convergence. The latter condition may be approached at every intensive bolt densities such as β>.30, which is not only rare in practice but is economically unattractive. The convergence ratio is particularly useful in the design of grouted and swellex bolts, since it reflects the reduction in convergence that can be achieved by a given bolt pattern.

An important characteristic of the convergence ratio is that it is insensitive to moderate changes of the deformation and strength parameters. For instance, a change in Young's modulus affects both ua^* and u_a equally, hence the ratio u_a^*/u_a remains unaltered. The latter characteristic of the normalized convergence ratio makes its use in design even more reliable, since the variation of *in-situ* geotechnical parameters can be tolerated without any significant error.

5.2.11. Concept of bolt effectiveness

In order to assess the efficiency of bolting, the opening convergence is selected as the appropriate evaluation criterion. Obviously, optimal efficiency of a bolt system corresponds to minimal opening convergence that can be achieved within economic limitations. In reality, the total convergence of a yielding, reinforced opening wall (u_a^*) must be less than of an unsupported opening (u_a), but more than the convergence predicted by the linear elastic solution (u_e). Considering these limitations the bolt effectiveness (i) for a given field stress is best defined as:

$$i = \frac{u_a - u_a^*}{u_a - u_e}\% \quad (101)$$

The bolt effectiveness (i) is sensitive to moderate changes in uniaxial compressive strength and the friction angle. Therefore, its use as a design tool is justified only if the geotechnical properties of ground are accurately determined.

5.2.12. Relationship between the analytical solution and rock mass classification approach (empirical design)

No analytical solution, according to engineering judgment, is valid on the condition that it is verified by empirical approaches. Rock mass classification is generally recognized the most suitable way of designing the reinforcement system in a rational manner.

Amongst rock mass classification systems, the Bieniawski's (RMR System) seems to be applicable to fully grouted bolts in all types of rock. The deficiency of the Q-system for defining an appropriate reinforcement system in the case of using the grouted bolt has been discussed earlier .Unfortunately, Bieniawski didn't offer any extra support design guideline apart from that has been published in the literature.

According to Bieniawski's guidelines (1974, 1979, 1989), the recommended bolt lengths (L) and grid spacing ($S_L \times S_T$) for different rock classes are tabulated in the first three columns of Table2. The ratio β/λ for these rock classes can be deduced from this information and is tabulated in the forth column. The magnitude of λ may be estimated from the effective bond angle of the bolt/grout interface to determine β. The corresponding bolts density parameters for an assumed $\lambda=0.5$ are given in the last column of Table 2. Several interesting aspects evolve from this table. The bolt densities (β) recommended for poor to very-poor rock are relatively insensitive to rock quality changes and the advocated range of β for some rock classes (RMR< 40) is very wide.

According to surveyed conducted, for RMR<20, bolt densities seems to be too lower. Based on results obtained from experiences, as the bolt density increases, so the spacing decreases. Hence a reduction of the bolt spacing for a weakest rock class would provide a sufficiently high magnitude for β to curtail displacements more effectively than by increasing the bolt length. This finding is so important in that in the weak rock mass RMR<40, for instance at the weak, stratified, clay bearing rock masses perfectly characterized by Ünal (1996), decreasing the bolts spacing is far more effective than increasing the bolts length. This evidence was also supported by Laubscher and Taylor who proposed bolt spacing less than 0.75m for poor ground at RMR<30. This bolt spacing corresponds to a β-value of about 0.28 for $\lambda=0.5$, and seems to be in good agreement with the densities proposed by Intraratna (1987,1990) for effective convergence reduction.

In my opinion, on the basis of results and observations, I concluded that the RMR system may not provide a sufficiently sensitive guide to properly designed grouted bolts, even all types of fictitious bolt, in weak, yielding rock mass. For classes of poor rock (RMR<40), a rational design method for grouted bolts should be based on an analytical approach, which provides a sound basis for effective convergence control. However, for reinforcement system guidelines fro RMR<40, it is wise to classify the rock mass based upon Modified Rock Mass Rating (M-RMR) developed by Ünal (1996) and then in order to better characterize the rock mass in terms of strength parameters, the M-RMR class of rock mass can be switched into GSI (Geological Strength Index), leading ultimately to a new concept of empirical reinforcement design, which satisfies analytical methods.

Table 2. Recommended bolt densities according to Geomechanics Classification (RMR) (*after* Intraratna & Kaiser, 1990)

Rock class		L	S_L and S_T		β
RMR	Condition	(m)	(m)	β/λ	(at
λ=0.5)					
81-100	Very good		no support		0.00
61-80	Good	2-3	2.5	0.05	0.10
41-60	Fair	3-4	1.5 – 2.0	0.08 – 0.14	0.04-0.07
21-40	Poor	4-5	1.0 – 1.5	0.14 – 0.31	0.07 – 0.16
<20	Very poor	5-6	1.0 – 1.5	0.14 - 0.31	0.07 – 0.16

5.2.13. Discussion and verification of the analytical model

The development of load on a grouted bolt has the effect of providing additional confinement (increased radial stress) in the yielded zone. As a result the tangential stress at the same point is increased more than proportionately. The original failure envelope is thereby shifted upwards, indicating an improvement of the apparent strength (σ_c, φ), as represented by the Mohr-Coulomb diagram in Figure 14. This enables the rock mass to behave as a stronger material leading to a corresponding reduction in opening convergence at a given field stress.

Owing to the fact that fully grouted bolts effectively improve the apparent strength of the rock mass, the behavior of the reinforced opening can be ideally represented by a shift of the ground convergence curve. The vertical axis of the ground convergence curve (Figure 15) represents the fictitious radial stress (σ_s) required at the opening boundary to prevent further convergence. The horizontal axis represents the opening convergence at the opening wall (u_a). The ground convergence curves are identical at every point along the opening boundary for the condition of axisymmetric yielding under hydrostatic field stress.

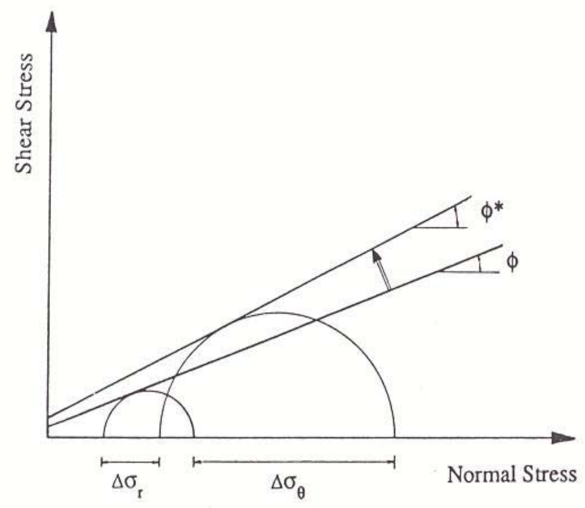


Figure 14. Effect of grouted bolts on failure envelope (*after* Intraratna, 1987,1990)

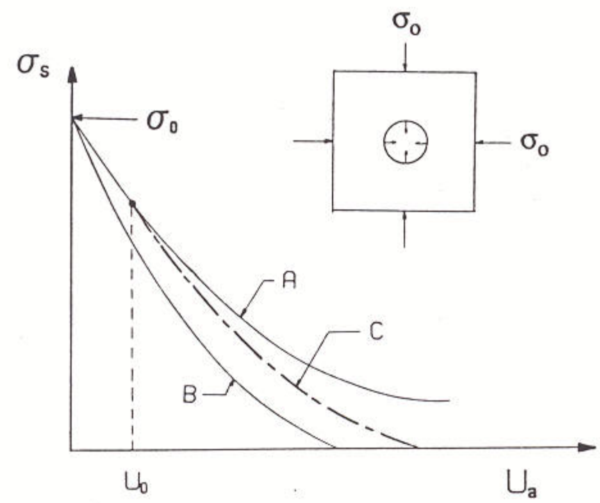


Figure 15. Effect of grouted bolts on ground convergence curve ((*after* Intraratna, 1987,1990)

The response of an unsupported opening in yielding rock is given by curve A. curve B represents an imaginary ground convergence curve of the opening, where bolts would have been installed before any displacements could have occurred. In reality, an initial displacement (u_o) of the opening wall occurs prior to the installation and subsequent activation of the grouted bolts. The magnitude of convergence after bolting is dependent on the apparent stiffness of the bolt/ground composite, and is reflected by a shift of the ground convergence curve from curve A to curve C, as a result of the reduced yield zone. In contrast to fully grouted bolts, pre-tensioned mechanical bolts provide direct radial pressure (active support) against the opening wall, but do not become an integral part of the deforming rock mass. Consequently, their performance is best represented by a support confinement curve with a specific stiffness and its interaction with the original ground convergence curve.

PART B: NON-LINEAR CONSTITUTIVE MODEL USING ASSOCIATED FLOW RULE AND EMPIRICAL HOEK & BRROWN YIELD CONDITION

In this part, constitutive models associated with passive rock-bolt systems are discussed based on non-linear Hoek and Brown failure criterion using associated flow rule. Unlike previous constitutive model, this model has been highly subjected to argue due to its complexity. Since the Ph.D. study is concerned with passive rock-bolt systems (grouted and swellex bolts) the Hoek and Brown solution (1980) corresponding to pre-stressed rock-bolt design won't be disputed here. Hoek and Brown (1980) introduced a solution containing ground reaction and available support curves (convergence-confinement method) for a circular opening at a hydrostatic field stress in a homogeneous, isotopic rock mass based on theory of elasticity. They took their non-linear failure criterion, associated flow rule, and a material behavior model prosecuting elastic-brittle –perfectly plastic into account. Based on their solution, alternatively recommended support systems could be utilized in a variety of rock mass conditions. Despite their sophisticated solution, no analytical model for passive rock-bolt system and its interaction with rock mass could have been presented. However, Hoek and Brown only pointed out that the support action of grouted rock-bolts would arise from internal reinforcement of the rock mass in much the same way as the presence of reinforcing steel acts in reinforced concrete. Up to that time, no direct evidence was available on the strength of reinforced rock masses.

Oreste and Peila (1995.1996,1997,2003) introduced a design of passive rock-bolt systems based on convergence-confinement concept. In my opinion, there have been some inconveniences and uncertainties in their solution; however, the model they used is so complex.

5.3.ORESTE & PEILA CONSTITUTIVE MODEL

Peila and Oreste (1995) presented a new convergence-confinement approach able to model a reinforced rock mass zone surrounding an opening zone with new improved properties.

A circular opening subjected to an undisturbed hydrostatic stress field, under plane strain conditions has been modeled (Figure 16). The rock mass has been considered homogeneous, isotropic, elasto-plastic with a strain-softening material behavior and non-linear Hoek and Brown yield condition both peak and residual parameters of failure criterion i.e. both m and s decrease from the peak value to the residual ones linearly with the tangential strain.

It is at this point so interesting that for the first time, the strain-softening model was chosen for modeling the reinforced rock mass around an opening. Brown et al (1983), however, in an outstanding article presented a new analytical solution for a unsupported circular opening following the previous solution of Hoek and Brown (1980) modeling the rock mass in the case of both elastic-brittle plastic and elastic-strain softening. The problem has been solved with the usual concepts applied in the converegnce-cofinement approach using a finite difference scheme (due to the mathematical complexity of the describing equation) because the differential equations that describe the rock mass stress-strain field, with and without bolts, have been easily solved by finite difference method (FDM). The mathematical solution follows the step-wise sequence of calculations for an elastic-strain, softening-plastic model in which post-peak dilatancy occurs at a lower rate with major principle strain in the constant strength plastic zone than in the strain softening zone. The bolts are considered to be fully bonded to the rock and with linear-elastic behavior. In this work it has been shown that small variations of the rock mass mechanical properties (the peak and residual strength parameters, the elastic modulus and the strain softening parameters) in the reinforced zone with respect to natural rock mass, influence the opening deformations and the stresses induced in the rock mass.

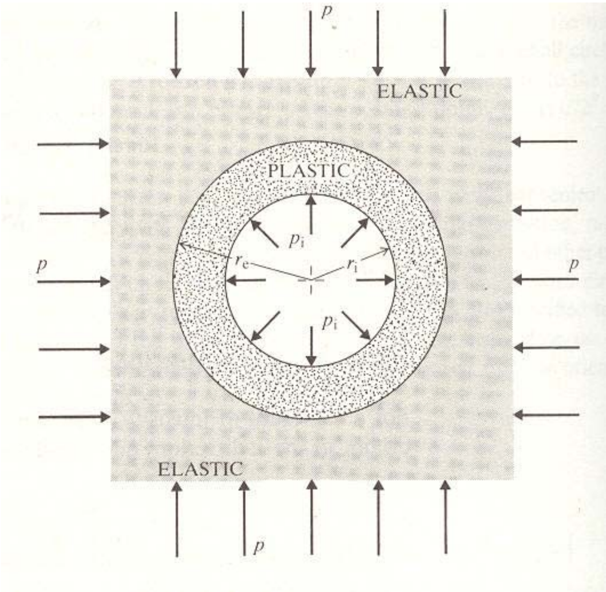


Figure 16. Assumed opening geometry (*after* Brady & Beown, 1985).

This model is capable of taking the effect of the distance from the bolted section to the opening face, the effect of increasing the lateral spacing between bolts (S_T in Fig.17) and the influence of different bolt end plate response curves into consideration.

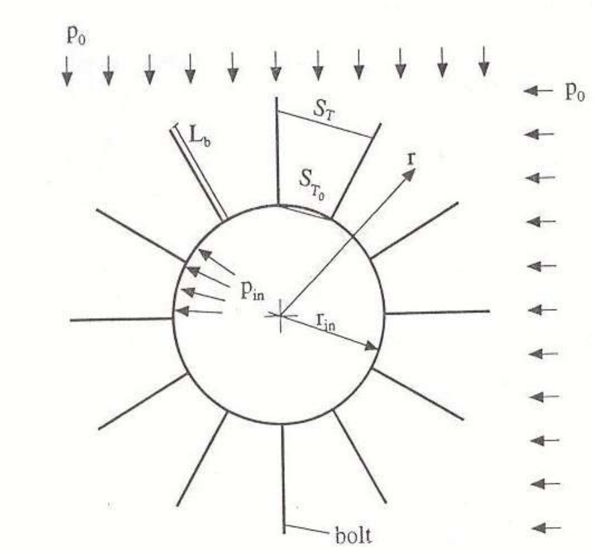


Figure 17. Geometry of the axsisymmetric problem (*after* Oreste & Peila,1996).

When the bolts are installed, a certain convergence of the natural rock mass has already developed (with respect to undisturbed condition). This is modeled by reducing the fictitious internal pressure from the P_o value to a percentage of this value (Panet and Guenot, 1982), according to the distance from the opening face; a further reduction of the internal pressure (simulation of the excavation process (FLAC 4, 2000)) induces tensile forces in the bolt which are applied to the rock mass through the bolt lateral surface and the nut end-plate. Indraratna & Kaiser (1990) had mentioned that relatively small displacements (4-5 mm) were normally sufficient to mobilize axial bolt tension by shear stress transmission from the rock to the bolt surface. The force applied by the bolt to the opening surface is strictly dependent on the nut end-plate stress-strain behavior. The condition of ideal tie, which means zero flexibility of the nut end-plate, allows the bolt to transfer the maximum force to the opening surface. In reality, however, even a lower bound condition where zero force is applied to opening perimeter is not taken to account and an intermediate condition is presented. The main steps of the analysis are as follows:

a. Computation of the convergence-confinement curve without bolts and stresses and strains for an internal pressure P_{in} value between P_o and the internal pressure of bolt installation (P_{inst}) [$P_{inst}<P_{in}<P_o$].
b. Computation of the convergence-confinement curve of a fictitious opening with a radius equal to the real opening radius plus the bolt length [$r_{fict}=r_{in}+L_b$];
c. Computation of the real stress-strain field influenced by the presence of the reinforcing elements starting from the values computed in step (b), for each point of the convergence-confinement curve of the fictitious opening (displacement and internal pressure), using a finite difference procedure. The value of the pressure and radial

displacement obtained at the opening radius are the real values of the bolted convergence-confinement curve.

The final convergence-confinement curve is therefore obtained by considering the values computed in step (a) for an internal pressure which varies from P_o to P_{inst}, and the curve computed in step (c) fro an internal pressure which varies from P_{inst} to zero.[$0<P_{in}<P_{int}$].

5.3.1.Rock mass behavior model

The strength criterion adopted is that proposed by Hoek & Brown (1980,2002):

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \quad (102)$$

where m_b is a reduced value of the material constant m_i and is given by:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (103)$$

s and a are constants for the rock mass given by the following relationships:

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \text{ and}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \quad (104)$$

D is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed *in-situ* rock masses to 1 fro very disturbed rock masses. Guidelines for the selection of D are shown elsewhere (Hoek et al. 2002).

It is assumed that the residual strength criterion is given the same analytical formulation, where the parameters m and s refer to the residual values m_r and s_r.

A strain softening behavior is assumed in the plastic zone (Brown et al.1983). The f and h parameters describe the magnitude of the plastic behavior after the peak condition (ratio between the radial and tangential plastic strain both in the residual and softening branch of the stress-strain curve), while α defines the width of the softening zone (see Figure 2). Experimental data are required to determine these parameters (Brown et al.1983, Peila & Oreste, 1995). The appropriate rock mass properties can be obtained by back-analysis of field data and are based on the designer experience.

Peila and Oreste (1985) presented an analytical expression of f using associated flue rule of the theory of plasticity corresponding to Hoek and Brown Failure Criterion:

$$f = \frac{m}{2\sqrt{m\sigma_r C_o^{-1} + s}} + 1 \quad (105)$$

Brown, Bray, Ladanyi, and Hoek (1983) presented, as earlier mentioned, an analytical solution for a circular opening in the case of strain-softening behavior of material which allows for the non-linearly empirical Hoek & Brown failure criterion. In that case, the solution must account for the possible existence of three different zones around the opening: (1) An elastic zone remote from the opening; (2) an intermediate plastic zone in which the stresses and strains fall on the strain-softening portion of Fig 2; and (3) an inner plastic zone in which stresses are limited by the residual strength of the rock mass (see Figure 18). For non-axisymmetric problems, comparable closed-form or iterative finite difference solutions cannot be obtained, and a numerical method of computation, such as the finite element method (FEM), must be used (Brady & Brown, 1985).

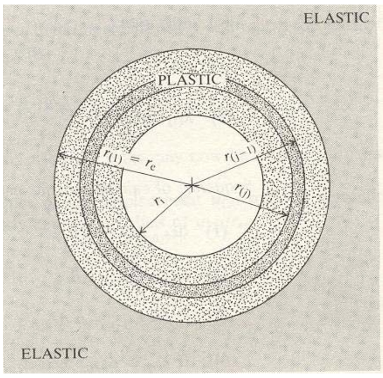


Figure 18. Plastic zone divided into thinannuli for finite difference solution (stepwise geometrical discretization) (after Brady & Brown,1985).

According to Figure 19, the zones around an opening can be defined based on the properties of the rock mass. An unsupported opening, $P_i=0$, constitutes two major rock mass around itself namely, broken (plastic or yielding) zone and elastic (natural) zone while a reinforced opening is embedded within reinforced rock mass and elastic zone. What is by far important to recognize is that how much the reinforced rock mass accounts for yielding zone. Intraratna (1987) regarded the reinforced rock mass surrounded within yielding zone as equivalent plastic zone (EPZ). The connection between those zones is of great importance in order to better understand the behavior of reinforced rock mass.

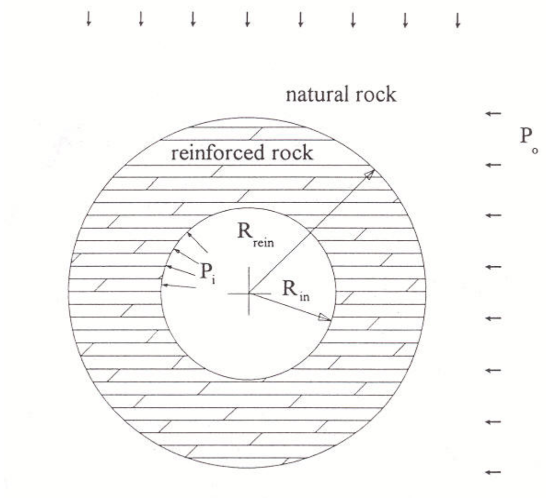


Figure 19. studid problem scheme (after Peila & Oreste, 1995).

With a assumption that a linear elastic stress-strain behavior occurs before failure and use the equilibrium equation (Equ. 51) and boundary conditions $u=0, \varepsilon_r=0, \varepsilon_\theta=0, \sigma_r= 0$ at $r=\infty$ and $\sigma_r= \sigma_{re}$ (σ_{re} is the radial stress at failure) at the plastic-elastic interface (r_e), stress and strain in the elastic region will be:

$$\sigma_r = P_o - (P_o - \sigma_{r(brk)})\left(\frac{r_e}{r}\right)^2 = \sigma_3 \quad (106)$$

$$\sigma_\theta = P_o + (P_o - \sigma_{r(brk)})\left(\frac{r_e}{r}\right)^2 = \sigma_1 \quad (107)$$

$$\varepsilon_r = -(P_o - \sigma_{r(brk)})\frac{1+\nu}{E}\left(\frac{r_e}{r}\right)^2 = \varepsilon_3 \quad (108)$$

$$\varepsilon_\theta = -(P_o - \sigma_{r(brk)})\frac{1+\nu}{E}\left(\frac{r_e}{r}\right)^2 = \varepsilon_1 \quad (109)$$

$$u_r = -(P_o - \sigma_{r(brk)})\frac{1+\nu}{E}\left(\frac{r_e^{\ 2}}{r}\right) \quad (110)$$

at $r=r_e$ the radial and tangential stresses (minor and maximum principal stresses) apart from having to satisfy the elasticity theory, must also satisfy the adopted failure criterion:

$$\sigma_{\theta(brk)} = \sigma_{r(brk)} + \sigma_c\left(m_b\frac{\sigma_{r(brk)}}{\sigma_c} + s\right)^a \quad (111)$$

where $\sigma_{\theta(brk)}$ is the tangential stress at failure.

Equation 111 may be rewritten, using equations 106 and 107:

$$2(P_o - \sigma_{r(brk)} = \left(m_b\sigma_{ci}\sigma_{r(brk)} + s\sigma_{ci}\right)^a \quad (112)$$

from which, after some simplifications, one obtains:

$$\sigma_{r(brk)} = P_o - M\sigma_{ci} \quad (113)$$

where

$$M = -\frac{m_b}{8} + 0.5\sqrt{\left(\frac{m_b}{4}\right)^2 + \frac{m_b P_o}{\sigma_{ci}} + s}$$ and a is considered to be 0.5. (114)

It is therefore possible to define the strains at the elastic-plastic interface, from equations 108, 109 and 113:

$$\varepsilon_{r(r_e)} = -M\sigma_{ci}\frac{1+\nu}{E} \quad (115)$$

$$\varepsilon_{\theta(r_e)} = M\sigma_{ci}\frac{1+\nu}{E} \quad (116)$$

The mathematical formulation proposed by Brown et al. (1983) has been utilized in order to describe the stress-strain behavior of the rock mass in the plastic (strain-softening) zone, It is assumed that both m and s decrease from peak values to the residual value m_r and s_r, linearly with the tangential strain:

$$m_{(j+1)} = m + (m_r - m)\frac{(\varepsilon_{\theta_{(j+1)}} - \varepsilon_{\theta_{(re)}})}{(\alpha-1)\varepsilon_{\theta_{(re)}}} \quad (117)$$

$$s_{(j+1)} = s + (s_r - s)\frac{(\varepsilon_{\theta_{(j+1)}} - \varepsilon_{\theta_{(re)}})}{(\alpha-1)\varepsilon_{\theta_{(re)}}} \quad (118)$$

where $m_{(j+1)}$ and $s_{(j+1)}$ are the m and s values in the softening zone at a point where the tangential strain is $\varepsilon_{\theta(j+1)}$.

On this basis, the value of r_e, the position of which has not yet been defined, is obtained with a numerical solution based on the finite difference method. The computational procedure involves the discretization of the plastic zone in annular rings starting from the unknown elastic-plastic interface towards the opening, by incrementing the tangential strain value $\varepsilon_{\theta(j)}$ for every calculation step and calculating the corresponding radial strain. Since, from the above equations, at $r=r_e$, the radial stress, m_b and s (peak value) are known, and by assuming that the tangential and radial strain are continuous the following can be written:

$$\sigma_{r_{(re)}} = \sigma_{r(brk)} = \sigma_{r(o)} \quad (119)$$

$$m_{(re)} = m_{(o)} \quad (120)$$

$$s_{(re)} = s_{(o)} \quad (121)$$

$$\sigma_{\theta_{(re)}} = \sigma_{r_{(re)}} + \sqrt{m_b\sigma_{ci}\,\sigma_{r_{(re)}} + s\sigma_c^{\;2}} = \sigma_{\theta_{(o)}} \quad (122)$$

$$\varepsilon_{r_{(re)}} = \varepsilon_{r_{(o)}} \quad (123)$$

$$\varepsilon_{\theta_{(re)}} = \varepsilon_{\theta_{(o)}} \quad (124)$$

while for ring *(j+1),* which lies between the radii $r_{(j+1)}$ and $r_{(j)}$, the strains at the boundaries are defined by:

$$\frac{\Delta u_{(j+1)}}{\Delta r_{(j+1)}}=\frac{u_{(j)}-u_{(j+1)}}{r_{(j)}-r_{(j+1)}}\cong 0.5\left(\left[\frac{du}{dr}\right]_{(j)}+\left[\frac{du}{dr}\right]_{(j+1)}\right)=0.5(\varepsilon_{r(j)}+\varepsilon_{r(j+1)})$$

which, since

$$\frac{u_{(j)}}{r_{(j)}}=-\varepsilon_{\theta_{(i)}} \quad \text{and} \quad \frac{u_{(j+1)}}{r_{(j+1)}}=-\varepsilon_{\theta_{(j+1)}},$$

can be written as:

$$\frac{\lambda_{(j+1)}}{\lambda_{(j)}}=\frac{r_{(j+1)}}{r_{(j)}}=\frac{2\varepsilon_{\theta(j)}-\varepsilon_{r(j)}-\varepsilon_{r(j+1)}}{2\varepsilon_{\theta(j+1)}-\varepsilon_{r(j)}-\varepsilon_{r(j+1)}} \quad (125)$$

where: $\lambda_{(j+1)}$ and $\lambda_{(j)}$: ratio between the radius $r_{(j+1)}$ or $r_{(j)}$, and the unknown value of r_e; the tangential strain at $r_{(j+1)}$, $\varepsilon_{\theta(j+1)}$, is imposed by assuming an arbitrary incremental value at $r_{(j)}$ is known.

The stresses at the boundaries, for ring (j+1), are defined by:

$$\frac{\Delta\sigma_{r(j+1)}}{\Delta r_{(j+1)}}=\frac{\sigma_{r(j)}-\sigma_{r(j+1)}}{r_{(i)}-r_{(j+1)}}\cong\frac{\sigma_{\theta(j+1),med}-\sigma_{r(j+1),med}}{0.5(r_{(j)}+r_{(j+1)})}$$
$$=\frac{\sqrt{(0.5m_{med(j+1)}.(\sigma_{r(j+1)}+\sigma_{r(j)})\sigma_{ci}+s_{med(j+1)}\sigma_{ci}{}^{2})}}{0.5(r_{(j)}+r_{(j+1)})} \quad (126)$$

where,$m_{med(j+1)}$,$s_{med(j+1)}$: average strength parameters. After some substitution equations 125, 126,and 128 allow one to define the radial and tangential stresses at the linear surface of ring *(j+1)*:

$$\sigma_{r(j+1)}=b-\sqrt{b^2-A} \quad (127)$$

where:

$$A=\sigma_{r_{(j)}}-4k(0.5m_{med(j)}\sigma_{ci}\sigma_{r(j)}+s_{med(j)}\sigma_{c}{}^{2})$$

$$b=\sigma_{r(j)}+km_{med(j)}\sigma_{ci}$$

$$k=\left(\frac{r_{(j)}-r_{(j+1)}}{r_{(j)}+r_{(j+1)}}\right)^2=\left(\frac{1-\lambda_{(j+1)}}{1+\lambda_{(j+1)}}\right)^2$$

$$\sigma_{\theta(j+1)}=\sigma_{r(j+1)}+\sqrt{m_{(j+1)}\sigma_{ci}\sigma_{r(j+1)}+s_{(j+1)}\sigma_{ci}{}^{2}} \quad (128)$$

By carrying out the iteration from the unknown value of the plastic radius and by calculating the value of $\lambda_{(j+1)}$ for each iteration, the plastic radius, for a defined internal pressure P_{in}, results to be:

$$r_{e(p_{in}=\sigma_{r(j+1)})} = \frac{r_i}{\sum_{j=0}^{n} \lambda_{(j+1)}} \quad (129)$$

where n is the total number of iteration. The problem is solved computing $\sigma_{r(j+1)}$, $u_{r(j+1)}$ and $r_{e(j+1)}$ for each calculating ring.

5.3.2. Reinforced rock mass and grouted bolt behavior simulation

When the reinforcing elements are to be accounted for, the equilibrium equation, as shown in Figure 20, must be rewritten as:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{\sigma_r} + \frac{dT}{dr}\frac{r_i}{S_L S_T}\frac{1}{r} \quad (130)$$

where:

S_L= longitudinal bolt spacing along the opening axis;

S_{To}= transversal bolt spacing on the opening perimeter;

r = distance between the considered point in the rock mass and the opening axis;

T = axial tensile force in the bolt at a distance r, from the opening axis;

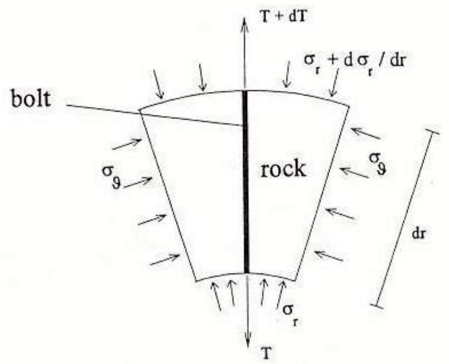


Figure 20. Stresses in the bolt and in the rock mass(after Oreste &Peila,199).

Assuming that the principal stress directions are not affected by the bolts, and that there is no adherence loss between the bolt and the rock mass, the axial force in the bolt (T) depends upon the radial strain in the rock mass, which is developed following bolt installation:

$$T = -E_b A_b (\varepsilon_r - \bar{\varepsilon}_r) \quad (131)$$

where ε_r is the radial strain in the rock mass at a distance r from the opening axis, ε_r is the radial strain in the rock mass at the same distance r when the bolt is installed, E_b is the Young's modulus of the bolt and A_b is the bolt transverse section.

The numerical calculation procedure used for the simulation of the plastic natural rock mass, described in the previous paragraphs, is now applied both in the elastic and plastic conditions. Starting from the known values of the convergence-confinement curve of the fictitious opening (at r_{fic}) the calculation rings develop towards the opening perimeter until rock failure occurs. Equations 106 to 110 are now substituted by equations 133 to 136, written in a finite difference form, for a distance $r_{(j+1)}$ from the opening axis. $r_{(j+1)}$ is defined by equation 132 and depends on the incremental value of the tangential strain.

$$r_{(j+1)} = \frac{-t + \sqrt{t^2 - 4 \cdot \xi \cdot z}}{2 \cdot \xi} \quad (132)$$

$$\sigma_{r(j+1)} = \overline{\sigma}_{r(j+1)} + q(\varepsilon_{\theta\,(j+1)} - \overline{\varepsilon}_{\theta\,(j+1)}) + c(\varepsilon_{r(j+1)} - \overline{\varepsilon}_{r(j+1)}) \quad (133)$$

$$\sigma_{\theta(j+1)} = \overline{\sigma}_{\theta(j+1)} + \frac{1}{e}(\varepsilon_{\theta\,(j+1)} - \overline{\varepsilon}_{\theta\,(j+1)}) + q(\varepsilon_{r(j+1)} - \overline{\varepsilon}_{r(j+1)}) \quad (134)$$

$$\varepsilon_{r(j+1)} = \overline{\varepsilon}_{r(j+1)} + \frac{r_{(j+1)}\left[2\left(\varepsilon_{\theta(j+1)} - \overline{\varepsilon}_{\theta(j+1)}\right) - \left(\varepsilon_{r(j)} - \overline{\varepsilon}_{r(j)}\right)\right]}{r_{(j+1)} - r_{(j)}} - \frac{r_{(j)}\left[2\left(\varepsilon_{\theta(j)} - \overline{\varepsilon}_{\theta(j)}\right) - \left(\varepsilon_{r(j)} - \overline{\varepsilon}_{r(j)}\right)\right]}{r_{(j+1)} - r_{(j)}} \quad (135)$$

$$u_{(j+1)} = \overline{u}_{(j+1)} - (\varepsilon_{\theta(j+1)} - \overline{\varepsilon}_{\theta(j+1)})r_{(j+1)} \quad (136)$$

where:

$$e = \left[\frac{1}{E} - \frac{(\nu + \nu^2)^2}{E^3}\right] - \frac{\left(\frac{\nu + \nu^2}{E}\right)^2}{\frac{1}{E_b} - \frac{\nu^2}{E}}$$

$$c = \frac{1}{\frac{1}{E_b} - \frac{\nu^2}{E}} + \frac{\left(\frac{\nu + \nu^2}{E}\right)^2}{\left(\frac{1}{E_b} - \frac{\nu^2}{E}\right)^2 e}$$

$$n = q - 2c; \quad q = \frac{\left(\frac{\nu + \nu^2}{E}\right)}{\left(\frac{1}{E_b} - \frac{\nu^2}{E}\right)e}; \quad \gamma = -2n + 2q - \frac{1}{e}$$

$$t = \frac{2}{e} r_{(j)}(\varepsilon_{\theta(j+1)} - \overline{\varepsilon}_{\theta(j+1)}) + 2nr_{(j)}(\varepsilon_{\theta(j)} - \overline{\varepsilon}_{\theta(j)}) + (2c - 2q)r_{(j)}(\varepsilon_{r(j)} - \overline{\varepsilon}_{r(j)}) +$$
$$2r_{(j)}\left[(\sigma_{\theta(j)} - \overline{\sigma}_{\theta(j)}) - (\sigma_{r(j)} - \overline{\sigma}_{r(j)})\right]$$

$$\xi = \gamma(\varepsilon_{\theta(j+1)} - \overline{\varepsilon}_{\theta(j+1)}) + n(\varepsilon_{r(j)} - \overline{\varepsilon}_{r(j)}) - (\sigma_{\theta(j)} - \overline{\sigma}_{\theta(j)})$$

$$z = r_{(j)}{}^2\left[2(\sigma_{r(j)} - \overline{\sigma}_{r(j)}) - \frac{1}{e}(\varepsilon_{\theta(j+1)} - \overline{\varepsilon}_{\theta(j+1)}) - 2q(\varepsilon_{\theta(j)} - \overline{\varepsilon}_{\theta(j)}) + q(\varepsilon_{r(j)} - \overline{\varepsilon}_{r(j)})\right] - (\sigma_{\theta(j)} - \overline{\sigma}_{\theta(j)}).$$

The over-lined variables in equations 132 to 136 refer to the stress-strain field in the rock-mass at the moment of the bolt installation, i.e. P_{in}=$P_{inst.}$.

The plastic radius in the reinforced rock mass (for internal pressures lower than the fictitious pressure P_{inst} is now known before the simulation of the plastic zone is started (i.e. the radius for which the tangential stress defined in the elastic field by equation 133 is greater than the tangential stress defined by the strength peak condition).

The axisymmetric equilibrium with bolts (Equ. 130) is expressed in a finite difference form for a generic ring by below equation:

$$\frac{\sigma_{r(j+1)}-\sigma_{r(j)}}{r_{(j+1)}-r_{(j)}}=\frac{\frac{1}{2}(\sigma_{\theta(j+1)}+\sigma_{\theta(j)})-\frac{1}{2}(\sigma_{r(j+1)}+\sigma_{r(j)})}{\frac{1}{2}(r_{(j+1)}+r_{(j)})}+\frac{T_{(j)}-T_{(j+1)}}{r_{(j)}-r_{(j+1)}}\times\frac{r_i}{S_L S_T}\times\frac{1}{\frac{1}{2}(r_{(j)}+r_{(j+1)})} \quad (137)$$

and from the Hoek and Brown failure criterion:

$$\frac{1}{2}(\sigma_{\theta(j+1)}+\sigma_{\theta(j)})-\frac{1}{2}(\sigma_{r(j+1)}+\sigma_{r(j)})=\sqrt{\frac{\overline{m}_a}{2}(\sigma_{r(j+1)}+\sigma_{r(j)})\sigma_{ci}+\overline{s}_a\sigma_{ci}{}^2} \quad (138)$$

one can obtain:

$$\frac{\sigma_{r(j+1)}-\sigma_{r(j)}}{r_{(j+1)}-r_{(j)}}=\frac{\sqrt{\frac{\overline{m}_a}{2}(\sigma_{r(j+1)}+\sigma_{r(j)})\cdot\sigma_{ci}+\overline{s}_a\sigma^2{}_{ci}}}{\frac{1}{2}(r_{(j+1)}+r_{(j)})}+\frac{T_{(j)}-T_{(j+1)}}{r_{(j)}-r_{(j+1)}}\cdot\frac{r_i}{S_T S_L}\cdot\frac{1}{\frac{1}{2}(r_{(j)}+r_{(j+1)})} \quad (139)$$

where

$$\overline{m}_a=\frac{1}{2}\left[m_{(j)}+m_{(j+1)}\right] \quad \text{and} \quad \overline{s}_a=\frac{1}{2}\left[s_{(j)}+s_{(j+1)}\right].$$

For the reinforced rock mass equation 126 is substituted by equation 139. Assuming the same geometrical discretization used for the solution of the stress-strain field in the natural rock mass for an internal pressure equal to P_{inst}, equation 127 is substituted by equation 140.

$$\sigma_{r(j+1)}=h-\sqrt{h^2-l} \quad (140)$$

where $h=\sigma_{r(i)}+k\overline{m}_a\sigma_{ci}-2k\eta$

$$l=\sigma^2{}_{r(j)}-4k\left[\frac{\overline{m}_a}{2}\sigma_{r(j)}\sigma_{ci}+\overline{s}_a\sigma_{ci}{}^2+\eta\sigma_{r(i)}-\chi\right]$$

$$\eta=\frac{(r_{(j)}+r_{(j+1)})(T_{(j)}-T_{(j+1)})r_i}{(r_{(j)}-r_{(j+1)})^2 S_L S_T}$$

$$\chi=\left[\frac{T_{(j)}-T_{(j+1)}}{r_{(j)}-r_{(j+1)}}\right]^2\left[\frac{r_i}{S_L S_T}\right]^2$$

having assumed the same previously described discretization of the natural plastic rock mass, the $r_{(j+1)}$ value is known for each calculation ring.

From the radial and tangential strains continuity it is then possible to rewrite equation 125 where the radial and tangential strains at the inner surface of the considered annulus (respectively, $\varepsilon_{r(j+1)}$ and $\varepsilon_{\theta(j+1)}$) are unknown. Since an increment of the tangential strain, in the plastic zone, produces an increment of the radial strain [equation (141a) in the plastic residual zone and equation (141b) in the softening zone], equation (125) can be solved as follows:

$$\varepsilon_{r(j+1)} = \varepsilon_{r(j)} - f(\varepsilon_{\theta(j+1)} - \varepsilon_{\theta(j)}) \quad (141a)$$

$$\varepsilon_{r(j+1)} = \varepsilon_{r(j)} - g(\varepsilon_{\theta(j+1)} - \varepsilon_{\theta(j)}) \quad (141b)$$

$$\varepsilon_{\theta(j+1)} = \frac{\lfloor 2\lambda_{(j)} + (\lambda_{(j+1)} - \lambda_{(j)} f \rfloor \varepsilon_{\theta(j)} + 2(\lambda_{(j+1)} - \lambda_{(j)})\varepsilon_{r(j)}}{2\lambda_{(j+1)} + (\lambda_{(j+1)} - \lambda_{(j)}) f} \quad (142a)$$

$$\varepsilon_{\theta(j+1)} = \frac{\lfloor 2\lambda_{(j)} + (\lambda_{(j+1)} - \lambda_{(j)} g \rfloor \varepsilon_{\theta(j)} + 2(\lambda_{(j+1)} - \lambda_{(j)})\varepsilon_{r(j)}}{2\lambda_{(j+1)} + (\lambda_{(j+1)} - \lambda_{(j)}) g} \quad (142b)$$

where f and g are the parameters in the plastic zone. When the radial strain $\varepsilon_{r(j+1)}$ at $r_{(j+1)}$ is known, it is possible to calculate the axial force in the bolt $T_{(j+1)}$ (Equ. 131) , the radial stress in the plastic reinforced rock mass (Equ.140) and the corresponding tangential stress. Therefore, the problem is completely solved.

6. CONCLUSIONS

In an effort to present a new constitutive model of rock-support interaction analysis, a comprehensive literature survey on that topic was conducted. To date, four constitutive model of rock-support interaction theory have been developed based on stress-strain behavior and yield condition disciplines. While Stille’s approach and Indraratna & Kaiser’s analysis are in conjunction with elastic-brittle plastic stress-strain relationship and linear Mohr-Coulomb failure Criterion, Hoek & Brown and Oreste & Prila analyses are governed by non-linearly empirical failure criterion. The rock-support solution undertaken by Hoek & Brown for prestressed rock bolts in the case of elastic-brittle-plastic model of material is disregarded in this study. For the first time, in order to develop an accurate and reliable roc-support interaction analysis, the elastic-strain softening behavior of rock taken into account by Oreste & peila, however, there are some uncertainties and inconveniences in their theory due to its complexity of mathematical operations.

The author suggests enhancing the Indraratna & Kaiser's solution because their model is simple and back analysis of some physical models and two-site application showed a good agreement between measured data and their solution. The following deficiencies of Indraratna & Kaiser's solution were recognized while comparing with the other models:

- Linear yielding criterion
- Elastic-brittle-plastic stress-strain law
- Same elastic modulus in the reinforced zone and in the natural rock mass
- Only the strength of rock mass is reduced to the residual value regardless of considering the internal friction angle of rock mass.

If Indraratna & Kaiser's constitutive model is mingled and modified with strain-softening behavior of material as well as non-linear yield condition, a competitively precise model with the minimum assumptions would be achieved.

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